CSE 396 Lecture Tue 2/21 Spr 2017 (1)

Regexp-to-NFA example:

$$((ab)^* + a^*)\cdot(ba + bb)^*$$

outermost op is + (union)

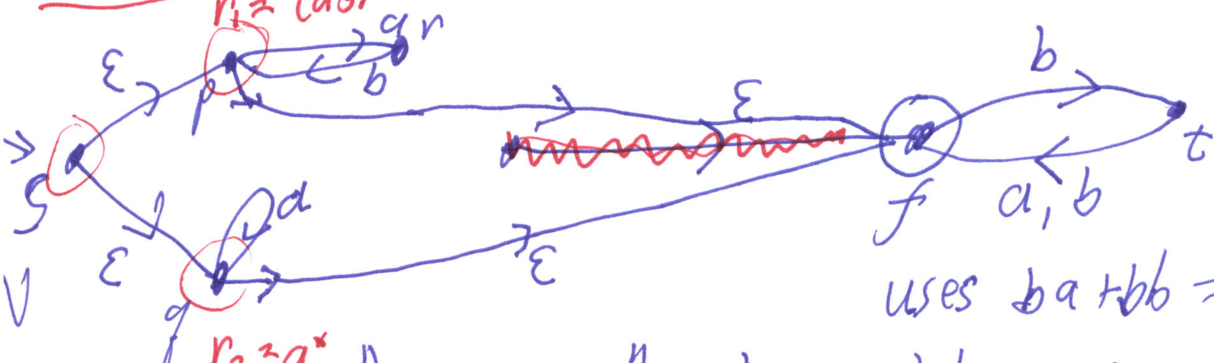
outermost op is •

outermost is *.

The whole expression parses as $(r_1 + r_2)\cdot(r_3^*)$

||

r_1 and r_2



r_3 — r_3 could be

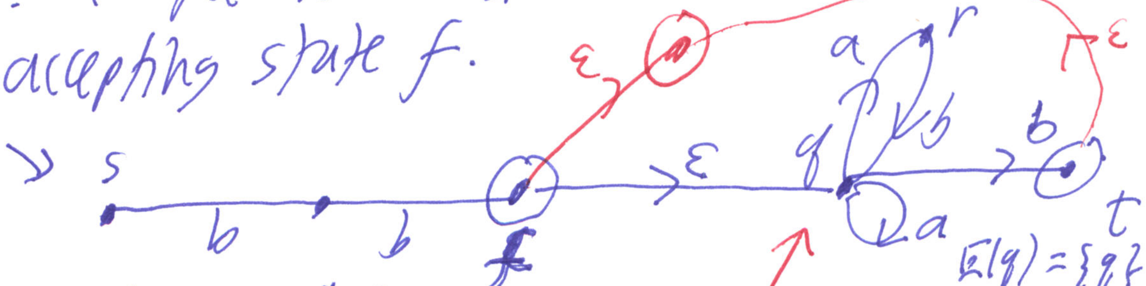
uses $ba + bb = b(a+b)$

$\boxed{E(s) = \{s, p, q, f\}.}$

"Dead strings": baa, bba, more strings of length ≥ 4

Does N accept ε? Yes because the Epsilon-closure E(s) includes the accepting state f.

N':



$E(q) = \{q\}$

$L(N') = bb(\varepsilon \cup (ab+a)^* b)$

does not allow bba

if we made this state accepting then we would incorrectly accept bba.

$E(f) = \{f, q\}$

Defn The ε-closure $E(p)$ of a state p is the set of all states reachable by zero or more ε-arcs out of p.

Theorem: For any NFA $N = (Q, \Sigma, \delta, s, F)$, we can build a DFA $M = (\mathcal{P}(Q), \Sigma, \Delta, S, \mathcal{F})$ s.t. $L(M) = L(N)$.

Abstract idea: States of M are subsets of possible states of N at any particular step i of processing an input $x = \underbrace{x_1}_{P} \cdots \underbrace{x_i}_{R} \cdots \underbrace{x_n}_{T}$.

S

is $T \in \mathcal{F}$?

Proof starts in the state $S = E(s)$.

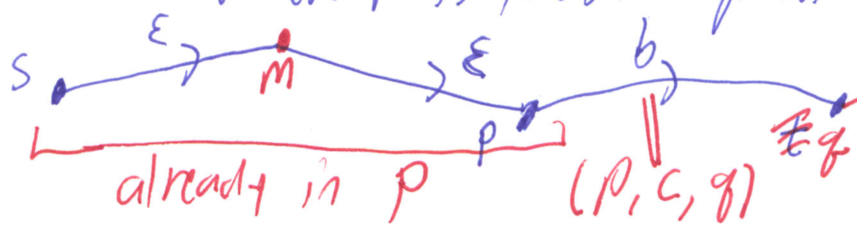
We will ensure that every intermediate state $P, R, \ldots$ and our final state T are also ε-closed.

Def^n: The ε-closure $E(P)$ of a set of states P is the union of $E(p)$ over all $p \in P$. $E(P) = \bigcup_{p \in P} E(p)$.

Proof: Build $S = E(s)$, and for any $P \subseteq Q$, $c \in \Sigma$:

$$\Delta(P, c) = E(\{q : \text{for some } p \in P,\ (p, c, q) \in \delta\})$$

Claim: For each i, the set R of states that N can be in after processing $x_1 \cdots x_i$ equals $\Delta(P, x_i)$ where P is the set of possible states after processing $x_1 \cdots x_{i-1}$.



$\Delta(\{s, m, p\}, b) = \{q, \underline{q'}\}$.

The claim is by induction. Base case $i=0$ holds since $S = E(s)$. For ind. hypothesis, suppose it holds for P, ie. $P = \{p : N$ can process $x_1 \cdots x_{i-1}$ from s to $p\}$.

We've defined $\underbrace{R}_{\Delta(P,c)} = E(\{q : \text{for some } p \in P, (p,c,q) \in \delta\})$ $\boxed{\text{where } c = x_i}$

We need to show

$R = \{r : N$ can process $x_1 \cdots x_i$ from s to $\underline{r}\}$.

how

(a) Let any $r \in R$ be given as defined. Then N can process $x_1 \cdots x_i$ from s to r.

(b) Suppose N can process $x_1 \cdots x_i$ from s to r. Show $r \in R$

Proof of (a): By def^n of $r \in R$, there is some state $p \in P$ [such that N can process $x_1 \cdots x_{i-1}$ from s to p] and instruction $(p,c,q) \in \delta$ such that $r \in E(q)$. Then N can process:

Ind. Hyp.

- $x_1 \cdots x_{i-1}$ from s to p (by Ind. Hyp).
- take the arc $p \xrightarrow{x_i} q$
- take 0 or more ε-arcs from q to r.

So: N can process $x_1 \cdots x_i$ from p to r.

Conversely (b): In the processing of $x = x_1 \ x_2 \cdots x_{i-1} \ x_i \cdots$

p q

there must have been some legal step $(p, x_i, q) \in \delta$

So $p \in P$, $(p, x_i, q) \in \delta$, and only ε-arcs were used from q to r.

Thus $r \in E(\{q : (p, x_i, q) \in \delta \text{ for some } p \in P\}) = \Delta(P, x_i)$.

This claim finally shows that the end state T of the DFA <u>equals</u> $\{r \in Q : N \text{ can process all of } x \text{ from } s \text{ to } r\}$.

By defn of <u>$x \in L(N)$</u>, N accepts x if and only if <u>some</u> such r is a final state, ie. $r \in F$.

Hence we want $T \in \mathcal{F}$ iff T includes some state in F.

So define $\mathcal{F} = \{T \subseteq Q : F \cap T \neq \emptyset\}$.

Then $L(M) = L(N)$. $\boxtimes$