CSE439, Fall 2024 Problem Set 2 Due Thu. 9/26, 11:59pm

Reading: For next week, read the rest of Chapter 5, read Chapter 6, and yes also read Chapter 7. You may find that Chapter 7 actually recapitulates a lot of stuff; for instance, I have already made the boxed point in section 7.6 about entanglement. Also read the Chapter 7 end notes, and for Yogi Berra, see https://yogiberramuseum.org/about-yogi/yogisms/.

——————*Assignment 2, due Thu. 9/26 "midnight stretchy" on CSE Autolab*——————

(1) Let $\mathbf{A} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & i & 0 & i \\ 1 & 0 & -1 & 0 \\ 0 & i & 0 & -i \end{bmatrix}$.

- Write $\mathbf{A}$ as a tensor product of two 2×2 matrices. (6 pts.)
- Let $\mathbf{x} = \frac{1}{2}[1, i, 1, i]$ be the quantum state vector from problem 3(a) on Assignment 1. Show how to compute $\mathbf{y} = \mathbf{H}^{\otimes 2}\mathbf{A}\mathbf{x}$ *without* doing any operations involving 4×4 matrices. (That is, you are not allowed to multiply $\mathbf{H}^{\otimes 2}$ and $\mathbf{A}$ together, nor apply $\mathbf{A}$ to $\mathbf{x}$. 12 pts., for 18 total. OK, you can do the direct 4×4 matrix/vector multiplications to check your work, but you must show the scratchwork for the way involving 2×2 matrices only.)

(2) Use the Wybiral circuit simulator https://wybiral.github.io/quantum/ (or similar), for two-qubit circuits. Give line 1 the **H T H** sequence discussed in lecture, which makes the probability of measuring 0 on line 1 (given input e_0 there) become the irrational number $\cos^2(\pi/8)$.

(a) There are four places to insert a **CNOT** gate whose control is on line 1. Find one such that on input $|00\rangle$, the probability of measuring 0 on line 1 is once again a rational number. (3 pts.)

(b) Then answer: Is the resulting state entangled? You can answer this without the kind of equational calculations done on problem (3) of Assignment 1. The principle is implicit in problem (1) here: one cannot entangle a separable state (nor unentangle an entangled state) by appending single-qubit operations only. (9 pts.)

(c) Now add Hadamard gates on line 2 before and after the target of the **CNOT** gate. Is the resulting state entangled? Also answer as best you can: why doesn't this violate the principle stated in (b)? (6 pts., for 18 total)

(3) Design three-qubit quantum circuits that given the all-zero basis state e_{000} as input create the states

(a) $\frac{1}{2}(e_{000} + e_{001} + e_{010} - e_{111})$,

(b) $\frac{1}{2}(e_{000} + e_{001} - e_{010} - e_{111})$, and

(c) $\frac{1}{2}(e_{000} - e_{011} + e_{100} + e_{111})$.

You may experiment and check your work with a quantum circuit simulator. Recall that multiplying everything by any unit scalar, in particular -1, gives the same quantum state. For a warmup, note that the 2-qubit circuit `H 1 H 2 CZ 1 2` applied to e_{00} produces the state $\frac{1}{2}(|00\rangle + |01\rangle + |10\rangle - |11\rangle)$. This is plain-text notation for the circuit with Hadamard gates on qubits 1 and 2 followed by a **CZ** gate between them. You are also welcome to use Dirac notation, so that e.g. the state in (c) is written $\frac{1}{2}(|000\rangle - |011\rangle + |100\rangle + |111\rangle)$. The circuit in (c) can be a graph state circuit, with **CZ** and **Z** gate(s) between an initial and final $\mathbf{H}^{\otimes 3}$. $(9 + 12 + 9 = 30$ pts., for 66 pts. total on the set)