The **Final Exam** is late in the exam period, on **Tuesday, Dec. 17**, 11:45–2:45pm. It has officially been moved from Baldy 111 (which was the original classroom in August) **to Fronczak 408**.

The Prelim II "extra problem" is not part of this set, but should be submitted in the same PDF. It is given on pages 12–13 of the posted notes https://cse.buffalo.edu/˜regan/cse439/CSE439Week14.pdf

——————-*Assignment 6, due Mon. 12/9 "midnight stretchy" on CSE Autolab*——————-

(1) Lipton-Regan text, exercise 14.7 on page 165: Use the spectral method to compute a square root of

$$\mathbf{W} = \frac{1}{\sqrt{2}}(\mathbf{X} + \mathbf{Y}) = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1-i \\ 1+i & 0 \end{bmatrix}.$$

(Please show all scratchwork. 18 pts.)

(2) Consider the 3-node "lollipop graph" G with a loop at node 1 and edges $(1, 2)$ and $(2, 3)$. Construct the corresponding graph state $|\Phi_G\rangle$ without the second bank of Hadamard gates.

(a) Construct the density matrix $\boldsymbol{\rho}_G$. Then show the result of tracing out node 3. Is the result a completely mixed state of two qubits? Then trace out node 2 as well and say what the first qubit is unto itself. (Yes, please do write out the 8×8 matrix of $+1$ and -1; you'll use it in part (d) too.)

(b) Now add a loop at node 3. Does this change the answers to part (a)? (Here and in (c) you may lean on *Quirk*, mindful of the little-endian display.)

(c) Call node 3 "Charlie," the others "Alice" and "Bob." Now let Charlie apply a gate that does not commute with **Z** and **CZ**—try the square-root-of-**Y** gate. (If you're curious to do this by hand as well as via *Quirk*, use the simplified form at the bottom of page 149 in section 14.6 of the text.) Does it matter now whether you place the $\mathbf{Y^{1/2}}$ before or after the **CZ** gate that involves Charlie?

(d) Now throw a CNOT gate from control on node 1 to target on node 3 after the **CZ** gate. On a density matrix, the action of a permutation is carried out both swapping rows and swapping columns—here swapping 5 and 6, then 7 and 8. Then trace out Bob and Charlie in one go, on paper. Is Alice at node 1 left with a mixed or pure state? $(12 + 3 + 6 + 9 = 30$ pts.)

(3) Calculate the full SVD of the matrix $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$. This does involve diagonalizing the 2×2 matrix $A^T A$. You are welcome to use an applet to check your work—mindful of some cosmetic differences noted in class. Please again show all scratchwork. (24 pts. total)

(4) Let G be the graph on four nodes with edges $(1, 2), (2, 3), (3, 4), (1, 4), (1, 3)$. Design the corresponding graph-state circuit. Then quantify the amount of entanglement if Alice holds qubits 1 and 2 wehile Bob holds 3 and 4. (Note that because the final Hadamard transform has only single-qubit gates, which do not affect entanglement, you can use the state before the final Hadamard transform rather than after it. 18 pts., for 90 on the set)