CSE439, Fall 2025 **Assignment 8** **Due Mon 12/8, 11:59pm**

The **Final Exam** is late in the exam period, on **Tuesday, Dec. 16**, 11:45–2:45pm. It will be held in the lecture room. It will have the same open-book, open-notes rules as the prelim exams, including the usage of notes on a laptop or Ipad-like device provided it is openly viewable in the front row of seats.

For these two problems, you are welcome to use a matrix computation app such as those linked in the Week 13 lecture notes, but you must show all the computational steps: A^*A or AA^*, the characteristic polynomial, diagonalizing, finding the vectors for V and the columns of U, and verifying $A = U\Sigma V^*$. Be aware that the applets differ in their limitations (to real numbers), size constraints, and order of doing things—in particular, differing from my notes. In problems where you truncate Σ to Σ^* by deleting one or more smallest singular values, go on to show $A' = U\Sigma' V^*$. You are also welcome to consult the answer key https://cse.buffalo.edu/ regan/cse439/CSE439F24ps6key.pdf from last year, whose problems (3) and (4) are parallel to these two.

——————-*Assignment 8, due Mon. 12/8 "midnight stretchy" on CSE Autolab*——————-

(1) Calculate the full SVD of the matrix $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 0 & 1 \end{bmatrix}$. This does involve diagonalizing the 2×2 matrix $A^T A$. You are welcome to use an applet to check your work, but please show all scratchwork. *Then* delete the smaller singular value to make a 3×2 matrix Σ' with only one nonzero entry, and finally compute $A' = U\Sigma' V^*$. Discuss briefly places where A' gives a decent approximation to A and places where it doesn't. (36 pts. total)

(*Meta-problem:* Instead of changing the first 1 in the second row of last year's matrix to a 2 as done here, change the upper-left 1 to a 2. Or change one of the 0s in last year's matrix to a 1. Observe the horror—and note that it cannot be blamed on quantum mechanics or exponentially large matrices this time.)

(2) Let G be the graph on four nodes with edges $(1,3), (2,3), (3,4), (2,4)$. [Note: 1 and 2 are *not* connected.] Design the corresponding graph-state circuit, *not including* the second Hadamard transform (note that since that transform has only single-qubit gates, it does not affect entanglement) and compute the resulting length-16 vector Γ.

(a) Quantify the amount of entanglement if Alice holds qubits 1 and 2 while Bob holds 3 and 4, by "rolling" Γ into a 4×4 matrix A and computing its rank. Here you may compute its rank directly or use the SVD—in the latter case, please give the singular values but you need not show any other scratchwork.

(b) Write out the 16×16 density matrix $\rho = |\Gamma\rangle\langle\Gamma|$ and then trace out Bob's two qubits at once. Note that this involves dividing ρ up into 4×4 quadrants and taking the trace of each quadrant. Does the resulting 4×4 density matrix ρ' represent a pure state? Is it a tensor product of two 2×2 matrices? Either way, are Alice's two qubits entangled? (18 + 12 = 30 pts., for 66 on the set)