CSE439, Fall 2026 **Problem Set 1** **Due Fri. 9/11, 11:59pm**

Reading: Next week's lectures will focus on the rest of Chapter 3 and also Chapter 4. I may also reference the larger Hadamard and Fourier matrices in sections 5.1–5.2 of Chapter 5 as examples, and also (while covering chapter 4), mention the fact in Section 5.3 that a Toffoli gate can simulate a NAND gate, so please peek ahead to them.

—————-*Assignment 1, due Fri. 9/11 "midnight stretchy" on CSE Autolab*—————-

(1) Put the following functions into order by asymptotic growth rate. In cases where $f(n) \sim Cg(n)$ for some constant $C > 0$, you may list f and g in either order, but most say what the constant C is. Here $\log_2$ is the logarithm to base 2. You must include explanations for your answers, but . Besides using L'Hôpital's Rule, taking logs of both sides or otherwise simplifying exponents may help. (You get one for free: the "missing item c" is any constant function c, and it comes fiirst. 24 pts. total for the rest)

(a) $a(n) = 2^{2\log_2 n}$.

(b) $b(n) = 2^{(\log_2 n)^2}$.

(d) $d(n) = 3n^2$.

(e) $e(n) = 2n^{\log_2 n}$.

(f) $f(n) = (\log_2 n)^{\log_2 n}$.

(g) $g(n) = (\log_2 n)^{\frac{\log_2 n}{\log_2 \log_2 n}}$.

(2) For each pair of objects **a** and **b** (which could be vectors or matrices), say which of the following are defined, and give the values if so:

(i) The inner product $\langle \mathbf{a}, \mathbf{b} \rangle$,

(ii) the ordinary (matrix) product $\mathbf{a} \cdot \mathbf{b}$,

(iii) the tensor product $\mathbf{a} \otimes \mathbf{b}$, and

(iv) the outer product $|\mathbf{a}\rangle \langle \mathbf{b}|$.

The superscript ^{T} means *transpose*, and in particular, converts a row vector into a column vector and vice-versa (without conjugating any complex entries). If **b** is a column vector, then $|\mathbf{b}\rangle$ means keep it that way, but $|\mathbf{b}\rangle$ means transpose it into a row vector and also conjugate any complex entries. ($4 \times 3 \times 3 = 36$ pts.)

(a) $\mathbf{a} = [0.6, 0.8, 0.8, 0.6]^T$, $\mathbf{b} = [0.6, 0.8]$;

(b) $\mathbf{a} = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$;

(c) $\mathbf{a} = [1, i]^T$, $\mathbf{b} = [i, 1]^T$;

(3) For each of the following objects (vectors or matrices) $\mathbf{c}$, say whether it can be written as a tensor product of smaller objects. If you say yes, find smaller objects $\mathbf{a}, \mathbf{b}$ such that $\mathbf{c} = \mathbf{a} \otimes \mathbf{b}$. If you saym no, prove that such $\mathbf{a}$ and $\mathbf{b}$ cannot exist. (All of them are unit vectors or unitary matrices, but the constant factors in front are not critical to the question so you may ignore them. $4 \times 6 = 24$ pts.)

(a) $\frac{1}{\sqrt{2}}[0.6, 0.8, 0.8, 0.6]$,

(b) $\frac{1}{2}[1, -1, -1, 1]$,

(c) $\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & -1 \\ 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$,

(d) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$,

(4) Challenge Question with Bonus: This question is worth 6 regular-credit points (making 90 total on the set) and up to 12 more extra-credit points.

Revisit cases in problem (2) where you said a product was *undefined*. For each one, say what you could possibly do to define it.

For example, suppose we have two column vectors $\mathbf{a}$ and $\mathbf{b}$ of lengths m and n where $m < n$. If m divides n, so $n = km$, then we could replicate $\mathbf{a}$ k times to make a length-n vector $\mathbf{a}'$ and then define $\langle \mathbf{a}, \mathbf{b} \rangle$ to be the same as the well-defined inner product $\langle \mathbf{a}', \mathbf{b} \rangle$. Part of the regular credit is for answering: if m equals 1, so that $\mathbf{a}$ is just a scalar a, then would you say $\langle \mathbf{a}, \mathbf{b} \rangle$ or the product $\mathbf{a}\c{d}ot\mathbf{b}$ is the same as just multiplying the vector $\mathbf{b}$ by the scalar a to get a vector, or should it be something else?

A possible source of extra credit is to try to make sense of things like $\langle x| \, |y\rangle \, |z\rangle$ where the vectors all have the same length m. You can well-define this by doing the inner product frist to get a scalar a that just multiplies the vector $|z\rangle$. But what if you try interpreting $|y\rangle \, |z\rangle$ as a tensor product first, getting a column vector $|w\rangle$ of length $n = m^2$. Then when you do $\langle x| \, |w\rangle$ you are in the case above where the factor k is m. So you replicate $|x\rangle$ m times and do the well-defined inner product of two vectors of length n. Do you get the same result? Similar in some way? Can we make products with Dirac notation be completely "freely associative"? (This allows some simple answers to get some extra credit, but I don't have ultimately definitive answers. If you happen to have done some similar things in machine learning where "tensor" objects get autofilled for compatibility, let me know.)