Name and St.ID#:____

CSE491/596, Fall 2021 First Prelim Exam Oct. 13, 2021

Open book, open notes, closed neighbors, 48 minutes. The exam totals **80** pts., subdivided as shown. *Show all work*—this may help for partial credit. Please do the problems *on these exam sheets.*

All notation is standard as in course readings and lectures; in particular, the reversal of string x is written x^R, with examples $011^R = 110$, $\epsilon^R = \epsilon$, and $x = x^R$ means x is a palindrome. the difference of two sets B and A is written $B \setminus A$; it also equals $B \cap \tilde{A}$ where $\tilde{A}$ stands for the complement of A.

(1) (24 pts.)

Over $\Sigma = \{0, 1\}$, define L to be the language of strings that have more occurrences of the substring 010 than occurrences of the substring 101. Prove using the Myhill-Nerode technique that L is not a regular language. (Note that occurrences of the substrings can overlap—but you can take that worry out of the picture by judicious choices of S and/or z.)

(2) (9 + 15 + 6 + 2 = 32 pts.)

Consider the regular expression $r = (aa + ba)^*(ab + bb)^*$ over the alphabet $\Sigma = \{a, b\}$.

(a) Design an NFA N with 4 states such that $L(N) = L(r)$.

(b) Design a DFA M such that $L(M) = L(r)$. You may either convert your NFA N from part (a) or design M by understanding r directly, but in the latter case you must have comments that explain the strategy. In particular, you must have at least one comment saying when the $(ab + bb)^*$ part comes into play.

(c) Does there exist a "kill string" u such that for all $z \in \Sigma^*$, $uz \notin L(r)$? Give a shortest such string if so.

(d) Convert your N into a 2-state generalized NFA G such that $L(G) = L(N)$. You need not follow the algorithm but can "just do it"—and this is worth 2 points mainly because 80 is congruent to 2 modulo 3.

(Space for Problem (2), continued...the last problem is overleaf)

(3) (24 pts. total) True/False *w*ith justifications: each question is 3 pts. for correct answer and 3 for an explanation (or definitive example or counterexample) in one or two sentences.

(a) If x is any string in the language $L(r)$ in problem (2), then its reversal x^R is also in $L(r)$.

(b) Every undecidable language is non-regular.

(c) If B is undecidable and A is decidable, then $B \setminus A$ must be undecidable.

(d) If B is non-regular and A is regular, *and* if $A \subset B$, then $B \setminus A$ must be non-regular.

End of Exam.