Name and St.ID#:____

CSE491/596, Fall 2021 Second Prelim Exam Dec. 1, 2021

Open book, open notes, closed neighbors, 48 minutes. The exam totals 80 pts., subdivided as shown. *Show all work*—this may help for partial credit. All notation is standard as in course readings and lectures. The third problem has a choice, (3a) XOR (3b). You may freely cite facts from lectures and notes, such as the languages A_{TM}, K_{TM}, and NE_{TM} being (c.e. but) undecidable and the languages D_{TM} and E_{TM} being (co-c.e. but) undecidable.

(1) (18 + 6 + 6 = 30 pts.)

Sketch on paper a deterministic Turing machine M that decides whether a given binary string x of length n is a palindrome using $O(\log n)$ space. The input x is given on a read-only input tape. It suffices to say what the value of each $O(\log n)$-size worktape of M represents and how it is updated (as and if needed) by the controlling loop(s).

Also state the running time of your M. Finally say how fast your machine could be if there were no restriction on the space. (Your answer can continue onto page 2.)

(2) $7 \times 2 + 3 + 3 = 20$ pts. *Short-answer questions:* Write your answers, yes/no or true/false, next to the corresponding question part. No justifications are needed (though as always, they could help for part credit).

(a) Which of the following classes are known to be different from polynomial space, i.e., PSPACE? Answer yes/no for each.

(i) P

(ii) NP

(iii) NL

(iv) $\mathsf{DSPACE}[O(n)]$ (also called DLBA)

(v) $\mathsf{NSPACE}[O(n)]$ (also called NLBA)

(vi) $\mathsf{NTIME}[O(n)]$

(vii) EXP

(b) *True/False?* If A is complete for NL under $\leq_m^{\log}$ and B is complete for P under $\leq_m^{\log}$, then $A \leq_m^{\log} B$.

(c) *True/False?* All NP-complete languages are known to be outside $\mathsf{DTIME}[O(n^2)]$.

(3) (30 pts. total)

Choice—Do EXACTLY ONE of the following two problems, (2a) XOR (2b). You must indicate clearly which you are attempting. Your reductions in either case **must** have sections labeled *Construction*, *Computability/Complexity*, and *Correctness.*

(3a) let L_2 be the language of the following decision problem:

INSTANCE: A program P in Java (or some other high-level programming language).

QUESTION: Are there two different inputs on which P halts and gives the same output?

(i) Prove by a mapping reduction that L_2 is undecidable.

(ii) Is L_2 c.e.? If you say yes, give a formal definition of L_2 using quantifiers and decidable predicates that justifies your answer. If not, sketch another reduction if your answer to (i) did not already have this consequence.

XOR

(3b) Prove that the language of the following decision problem is NP-complete, using a polynomial-time mapping reduction from 3SAT for the hardness part:

DUNGEONS AND RAYGUNS

INSTANCE: An undirected graph $G = (V, E)$, which you can picture as a network of underground rooms connected by dark narrow tunnels, and a number $k \geq 1$.

QUESTION: Can you fire k rayguns, one in each of k tunnels, so that every room gets hit? Here, when a gun is fired in a tunnel edge (u, v) in the direction of v:

- The room v is hit directly.
- Each room that v is connected to is hit indirectly.
- That includes the room u as being hit too (figure the gun was fired from u toward v).