CSE596, Fall 2022 **Problem Set 6** **Due Tue. Nov. 22**

A reminder that **Prelim II** will be on **Wednesday, November 30** *in class period*. It will cover assignments through this one.

The status of the lecture slated for next Monday, November 21, will be determined on Thursday.

The two weeks after Thanksgiving will be back on the normal schedule, with lectures MWF 4-4:50pm in O'Brien 112.

Reading:

Thursday's lecture will finish proving the PSPACE-completeness of the TQBF problem from ALR chapter 28, section 5. We have already shown that GAP is complete for NL under $\leq_m^{\log}$, and it will be quick to observe that the Circuit Value Problem (CVP) is complete for P under $\leq_m^{\log}$. The remaining pieces covered Friday and next Monday will be: oracle Turing machines and polynomial-time Turing reductions (ALR ch. 28 calls them "Cook reductions"), classical probabilistic computation, and the class BPP. All of these topics are in the last sections 17–19 of Debray's notes, though not in that order. Classical probability will be the springboard for the last five lectures on quantum computation.

Over Thanksgiving weekend (when there will be no other homework given before the exam), please read the following excerpts from my textbook *Introduction to Quantum Algorithms Via Linear Algebra* with Richard Lipton:

- https://cse.buffalo.edu/˜regan/cse491596/LRQmitbook2pp3-106.pdf up through ch. 8;
- https://cse.buffalo.edu/˜regan/cse491596/LRQmitbook2pp131-147.pdf up through section 14.4.

——————*Assgt. 6, due Tue. 11/22 "midnight stretchy" on CSE Autograder*——————-

(1) For each of the following stated relationships between complexity classes, say whether it is known to be true or not. In all cases where it is "known," you can prove it by applying theorems relating time and space complexity classes. Where you say "not known," explain why the theorem comparing the two complexity measures involved fails to yield the stated relationship. Note that $\mathsf{E} = \mathsf{DTIME}[2^{O(n)}]$ and $\mathsf{EXP} = \mathsf{DTIME}[2^{n^{O(1)}}]$. (Yes, you may consult the similar problem given last year—its key will give an idea of how much justification to give as well. $5 \times 6 = 30$ pts.)

(a) $\mathsf{DSPACE}[(\log n)^2] \subseteq \mathsf{P}$.

(b) $\mathsf{PSPACE} \subseteq \mathsf{EXP}$.

(c) $\mathsf{NP} \subseteq \mathsf{E}$.

(d) $\mathsf{NP} \subseteq \mathsf{DSPACE}[n^2]$.

(e) $\mathsf{NTIME}[O(n)] \subseteq \mathsf{E}$.

(2) Prove that for any $k \geq 0$ and $\epsilon > 0$, $\mathsf{DTIME}[n^k]$ is properly contained in $\mathsf{DTIME}[n^{k+\epsilon}]$. Show the numerical estimates needed for the conditions of the deterministic time hierarchy theorem to apply. (12 pts.)

(3) Show, with reference to 2(d) above, that NP cannot be equal to $\mathsf{DSPACE}[n^2]$. Use the fact that QBF is complete for PSPACE under $\leq_m^p$, the space hierarchy theorem, and the closure of NP under $\leq_m^p$. (12 pts.)

(4) Consider the following decision problem:

EDGE-DISJOINT PATHS
INSTANCE: A directed graph G and nodes $s_1, s_2, t_1, t_2 \in V(G)$.
QUESTION: Do there exist a path P_1 from s_1 to t_1 and a path P_2 from s_2 to t_2 such that no edge connects a vertex used in P_1 to a vertex used in P_2?

(a) Prove that this problem is NP-complete, using a reduction from 3SAT. (*Hint:* The standard "rungs and clause gadgets" architecture can be made to work and will combine elements seen in lectures and assignments. Don't forget to show its language belongs to NP. 30 pts. total)

(b) State and justify one or more "drastic" consequences if the language of this problem were to belong to NL like the GAP language does. (6 pts., for 36 ont eh problem and 90 on the set)