CSE596, Fall 2022 **Problem Set 7** **Due Wed. Dec. 14**

Reading:

For Friday's last lecture, please read the Lipton-Regan text, chapters 8 and also the short chapter 9. It is possible I may say a few words about Simon's Algorithm in chapter 10 but it will not be covered in detail. Again the link is

https://cse.buffalo.edu/~regan/cse491596/LRQmitbook2pp3-106.pdf up through ch. 8;

This is the last regular problem set. The quantum problems (2)–(6) are intended to be short exercises reviewing linear algebra in this new context. The "safety net assignment 8" is intended only to compensate for a poor final exam, originally because the 2018 exam was on the first Monday and was impacted by a snow-postponed major project deadline, and because the 2020 final was given online under less-known circumstances. It will not be for extra credit or to compensate for anything else.

——————*Assgt. 7, due Wed. 12/14 "midnight stretchy" on CSE Autograder*——————-

(1) Let an undirected graph $G = (V, E)$ have distinguished nodes s_1, s_2, t_1, t_2. Initially we place a chess queen on s_1 and a chess king on s_2. If the king and queen are ever on nodes connected by an edge, then the king is in check. The question is, can they execute a sequence of moves "asynchronously"—meaning either can move at any time and can make multiple moves in a row, not strict alternation or "lockstep" moves—so that the queen reaches t_1, the king reaches t_2, and the king is never in check?

Your task is to determine whether this decision problem belongs to NL or is NP-complete. Note that if the king and queen move simultaneously in lockstep, then this is the problem on Prelim II which is in NL. But if the queen could be "superposed" anywhere along its path to s_2 at once, so that the king would need to find a path that gives zero risk of being checked from the queen's path, then this becomes the same as the "Edge Disjoint Paths" problem which we saw to be NP-complete. So which problem is this one like? You must prove your answer, at least to the standard of the part of the Prelim II key that is relevant to your answer. (Trying to look this up on the Internet, even if you think you are just trying to learn in a supplementary manner to class notes, is forbidden. You have had plenty of notes and experience to work on this problem by thinking for yourself. 24 pts.)

(2) Compute the tensor product of the row vector $u = \frac{1}{\sqrt{2}}(1, i)$ with the column vector $v = \frac{1}{\sqrt{2}}\binom{1}{i}$. You will get a 2×2 matrix A. Does it matter whether you do the tensor product as $A = u \otimes v$ or as $A = v \otimes u$? Also answer: Is A unitary? (15 pts. total)

(3) Now change u to be the row vector $\frac{1}{\sqrt{2}}(1, -i)$, keeping v the same. Now does $u \otimes v = v \otimes u$? Is the 2×2 matrix you get, either way, unitary now? (15 pts. total)

(4) Now let A be any 2×2 matrix and A^2 its square under ordinary matrix multiplication (not tensor product). Does $A \otimes A^2$ always equal $A^2 \otimes A$? Try it when A is the Hadamard matrix, and diagram a little two-qubit quantum circuit to interpret what $A \otimes A^2$ and/or $A^2 \otimes A$ winds up being in this case. (15 pts. total)

(5) Show that the vector $w = \frac{1}{2}(1, 1, 1, -1)$ cannot be written as a tensor product of two smaller vectors. That is, it represents an entangled quantum state. Show this by writing out the equations you get if $w = (a, b) \otimes (c, d)$ and proving that they cannot be solved for this w. (You can if you want ignore the $\frac{1}{2}$ factor in w. 12 pts.)

(6) Let C_1 be the two-qubit quantum circuit consisting of one Hadamard gate on line 1, then a CNOT gate with control on line 1 and target on line 2, and then another Hadamard gate on line 1. Let C_2 be the circuit that looks like C_1 "upside down": it has Hadamard on line 2, then CNOT with control on line 2 and target on line 1, and finally another Hadamard gate on line 2.

(a) Draw these two quantum circuits. (3 pts.)

(b) Use matrices to show that these circuits are equivalent. (OK, multiplying 4×4 matrices is tedious but this is "good for you." Good for 12 pts., anyway)

(c) Draw the "maze diagrams" for these two circuits, and trace using "signed mice" the result of running each on the input state $|10\rangle$. Check that you get the same results, as part (b) mandates. (12 pts., for 27 on the problem and 108 on the set)