Name and St.ID#:____

CSE491/596, Fall 2022 Prelim I Oct. 12, 2022

Open book, open notes, closed neighbors, 48 minutes. The exam totals 80 pts., subdivided as shown. Do em all three problems on these exam sheets—there is no "choice" option. em Show your work—this may help for partial credit.

Notation: The alphabet Σ is $\{a, b\}$ for problem (2) but $\{0, 1\}$ for problem (3). For problem (1) it does not matter. The length of a string x is denoted by $|x|$. The complement of a language A is denoted by $\tilde{A}$ and equals $\Sigma^* \setminus A$, where $\setminus$ means difference of sets. Given sets A and B, the *symmetric difference* $A \bigtriangleup B$ is the same as $(A \cap \tilde{B}) \cup (B \cap \tilde{A})$, and also the same as $(A \setminus B) \cup (B \setminus A)$. In the way that union is like OR, it corresponds to the logical exclusive-or (XOR) operation.

(1) (5 × 4 = 20 pts. total) *True/False*. Please write out the words `true` and `false` in full. Brief justifications are not necessary but may help for partial credit.

(a) If A and B are regular, then $A \triangle B$ is always regular.

(b) If A and B are decidable, then $A \triangle B$ is always decidable.

(c) If A and B are computably enumerable, then $A \triangle B$ is always computably enumerable.

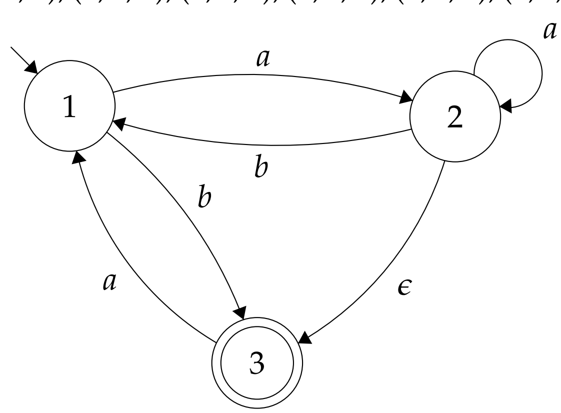
(d) If A is regular, then A^* is decidable in linear time by a single-tape Turing machine.

(e) Every non-regular language is decidable.

(2) 18 + 12 = 30 pts.

Consider the following nondeterministic finite automaton $N = (Q, \Sigma, \delta, s, F)$ where $Q = \{1, 2, 3\}$, $\Sigma = \{0, 1\}$, $s = 1$, $F = \{3\}$, and the instructions in δ are:

$$\{(1, a, 2), (1, b, 3), (2, a, 2), (2, b, 1), (2, \epsilon, 3), (3, a, 1)\}.$$



Convert N into a DFA M such that $L(M) = L(N)$ (18 pts.). Use the facing page for work. Also answer the following questions (3 pts. each).

(a) Is there a string u such that for each of its states q, N can process u from 1 to q? Give a shortest such string if so.

(b) Is there a string v that N cannot process starting from state 1 at all? Again give a shortest such string if so.

(c) Is there a string w such that for all $y \in \Sigma^*$, $wy \in L(N)$? Again give a shortest w if so.

(d) Does $L(N)$ include $b(aaabb)^*$? Briefly justify from your M.

(2) (workspace)

(3) (8 + 4 + 18 = 30 pts.)

Define L to be the language of strings x such that $|x|$ is even and the second half of x contains at least one '1.' For instance 010100 is in L but 01010000 is not, and 0100001 is not because its length is odd.

(a) Which of the following strings belong to L? Say yes/no for each.

$$(i)\ \epsilon \qquad (ii)\ 1 \qquad (iii)\ 01 \qquad (iv)\ 010.$$

(b) Is $L \cdot L \subseteq L$? Justify your answer briefly.

(c) Prove via the Myhill-Nerode Theorem that L is nonregular. (END OF EXAM, but fine to put any spillover work on problem (2) below, besides your work on this problem.)