CSE491/596 F23 **Problem Set 2** **Due Fri. 9/29 11:59pm**

This week moves on to Turing machines, computability, and undecidability. Before we get there, notice that the Fri. 9/22 lecture mentioned the DFA minimizatioin algorithm, which is the subject of Debray's section 4, but said to skip over it. Regarding the Myhill-Nerode coverage in section 5, we will also skim over the connection to streaming algorithms there and in section 6 at least for now. That brings us to Debray's section 7. After reading it, please jump way ahead to section 13 on page 39, but only as far as Theorem 13.4 atop page 41. The impression you should gain is that the two-tape Turing machines treat data more like a stream in an informal sense.

(1) Design a deterministic finite automaton $M = (Q, \Sigma, \delta, s, F)$ with $\Sigma = \{0, 1\}$ such that $L(M)$ equals the language L of binary strings x such that the number of *leading* 0s in x, plus the number of 1s in x, is even. Note that the empty string belongs to L, and if x begins with a 1, then it likewise is considered to have zero leading 0s.

Then execute the DFA-to-regexp conversion algorithm on your M. Note that because your M will have more than one accepting state that is different from the start state, you can't run the simplest form of the algorithm, but you can either add a new unique final state with ϵ-arcs from the previously final states, or you can break M into cases where just one state (different from start) is accepting and + together the results. (15 + 15 = 30 pts.)

(2) Prove that the following three languages over the alphabet $\Sigma = \{0, 1\}$ are non-regular, via Myhill-Nerode arguments. Here $\#c(x)$ denotes the number of occurrences of the character c in the string x, and for binary strings x, y of the same length, $x \oplus y$ is the bitwise exclusive-or. For example, $1011 \oplus 1101 = .0110$.

(i) $L_1 = \{x : \#0(x) \leq \#1(x)\}$.

(ii) $L_2 = \{x0y : |x| = |y|$.

(iii) $L_3 = \{xy : |x| = |y| \wedge x \oplus y = 1^{|x|}\}$.

($3 \times 12 = 36$ pts., for 66 total on the set)