# CSE491/596, Fall 2023 Problem Set 5 Due Fri. 11/10, 11:59pm

Lectures this week finish some leftover remarks from ALR sections 27.4–27.7 and then go to ALR chapter 28, section 5 on completeness for PSPACE, NL, and P under log-space reductions. The remaining pieces covered Friday and next Monday will be: oracle Turing machines and polynomial-time Turing reductions (ALR ch. 28 calls them "Cook reductions"), classical probabilistic computation, and the class BPP. All of these topics are in the last sections 17–19 of Debray's notes, though not in that order. Classical probability is the springboard for quantum computing. Please read Chapters 1–3 of the Lipton-Regan text over next weekend. The covered chapters—plus the first half of a later chapter—are available via non-public links on *Piazza*:

The **Second Prelim Exam** will be on **Wednesday Nov. 29** in class period. It will cover all the non-quantum material. There will be an Assignment 6 before it, which will have a mix of "classical" complexity and some elementary linear algebra exercises with reference to quantum complexity.

——————-*Assignment 5, due Fri. 11/10 "midnight stretchy" on CSE Autolab*——————-

**(1)** Give a polynomial-time many-one reduction from EXACTLY ONE 3SAT to the following problem:

**Instance:** A system of **linear** equations of the form $\ell_1(x_1, \dots, x_n) = 0$, $\ell_2(x_1, \dots, x_n) = 0$, $\dots$, $\ell_m(x_1, \dots, x_n) = 0$. (The linear forms $\ell_j$ are allowed to have nonzero constant terms, so they are really what is called "affine linear.")

**Question:** Does the system have a solution in which all the $x_i$ are 0 or 1?

*Then* write a short essay answer on this topic: "*Wait a second*—linear equations $\mathbf{Ax} = \mathbf{b}$ are solvable in polynomial time by Gaussian elimination. That goes for affine linear equations as well—the constants just get absorbed into the target vector $\mathbf{b}$. So why doesn't all this show $\mathsf{NP} = \mathsf{P}$?" Your answers should first observe that generally in this reduction we have $m > n$. Then notice the "incidental" point in the first problem of Assignment 4. You can even conclude that the above statement of the problem gives a misleading impression, though in fact the **Question** is perfectly legitimately worded. (A hint for the first part is to use the idea that OR is like addition, though that didn't work on Assignment 4. 12 + 12 = 24 pts.)

**(2) (24 pts.)**

Show that the following problem is NP-complete—by reduction from 3SAT or any of its variant forms which we have covered: NAE-3SAT, EXACTLY ONE 3SAT, or EXACTLY TWO 3SAT (your choice):

PERFECT DOMINATING SET
INSTANCE: An undirected graph $G$, an integer $k \geq 1$.
QUESTION: Is there a set $S$ of at most $k$ nodes such that every other node is adjacent to *exactly* one node in $S$?

*Note:* This is not the same as the standard DOMINATING SET problem. In a 3-node triangle graph, any one node forms a perfect dominating set, but the 5-node pentagon graph does not have any perfect dominating set of size 2. To earn full credit, you need to do a full job on the *Correctness* proof with separate $\Longrightarrow$ and $\Longleftarrow$ implications—you can't just draw a picture and wave your arms. (And don't forget to show the problem belongs to $\mathsf{NP}$. For 24 pts. extra credit, make the reduction work from "vanilla" 3SAT—this may involve "clause gadgets" a little bigger than the ones for the "vanilla" reduction to GRAPH 3-COLORING in ALR section 28.4 which was cut off in lecture by the fire alarm.)

**(3)** For each of the following stated relationships between complexity classes, say whether it is known to be true or not. In all cases where it is "known," you can prove it by applying theorems that were covered. Where you say "not known," explain why the theorem comparing the two complexity measures involved fails to yield the stated relationship. Note that $\mathsf{E} = \mathsf{DTIME}[2^{O(n)}]$ and $\mathsf{EXP} = \mathsf{DTIME}[2^{n^{O(1)}}]$.

(a) $\mathsf{NSPACE}[(\log n)^2] \subseteq \mathsf{P}$.

(b) $\mathsf{PSPACE} \subseteq \mathsf{E}$.

(c) $\mathsf{NP} \subseteq \mathsf{EXP}$.

(d) $\mathsf{P} \subseteq \mathsf{DSPACE}[n^2]$.

(e) $\mathsf{NSPACE}[O(n)] \subseteq \mathsf{E}$.

($5 \times 6 = 30$ pts. total, making 78 regular-credit points on the set)