CSE491/596, Fall 2023 **Problem Set 7 (the last)** **Due Sat. 12/9, 11:59pm**

Reading and Schedule

Lectures next week will focus on the applications in chapters 8–10 of the Lipton-Regan text. Monday 12/11 will illustrate the quantum part of Shor's algorithm from chapter 11 and show where this puts bounded-error quantum polynomial time (BQP) on the complexity map.

The **Final Exam** is on **Wednesday, Dec. 13** *in the lecture room, Norton 218*, from **3:30pm to 6:30pm** (not the classtime). It will have the same open-notes rules as the prelim exams. More so than the prelims, it will have some problems that combine different aspects of the course and/or allow more time for reflection and creativity. I've scheduled an in-person *Review Session* on the reading day, **Tue. Dec. 12**, starting at **10:30am** and going to 12pm or so. It will be **hybrid** and recorded. I will also have private office hours 1–2pm, but from 2pm onward is the CSE Awards Party in the Davis ground-floor atrium.

——————-*Assignment 7, due Sat. 12/09 "midnight stretchy" on CSE Autolab*——————-

(1) Design quantum circuits that given the all-zero basis state as input create the following states—note the extra minus sign in the second one:

(a) $\frac{1}{2}(|000\rangle + |001\rangle + |010\rangle - |111\rangle)$, and

(b) $\frac{1}{2}(|000\rangle + |001\rangle - |010\rangle - |111\rangle)$.

You may check your work with an online quantum circuit simulator—recall that multiplying everything by any unit scalar, in particular -1, gives the same quantum state. $9+9=18$ pts.

(2) Design a 4×4 unitary matrix U such that $U|00\rangle$ equals the state $|\phi\rangle = \frac{1}{\sqrt{3}}(|00\rangle + |01\rangle + |10\rangle$. For full credit, make U Hermitian as well as unitary. (12 pts. Then for a possible 6 pts. extra credit, say why one *cannot* design a circuit computing this U exactly from the basic gates we have seen so far in the course. A vocabulary word that might help you about the latter is, "dyadic.")

(3) Lipton-Regan text, exercises 4.11 and 4.12, OK to skip the "argue generally" part of the latter. (Note the references to the swap gate in problem 4.8, which was also sketched in lecture; 18 pts. total)

(4) Lipton-Regan text, exercise 8.3 on page 96. That is, for each of the other basis vectors $|00\rangle$, $|10\rangle$, and $|11\rangle$, show the spread of results from the four ways to fill the middle slot. Do any of those vectors yield a similarly easy way of distinguishing between "constant f" and "non-constant f"—by measuring one of the qubits—as the choice $|01\rangle$ did? (If you catch on that one of the three choices is going to be redundant with work already done, you can abbreviate it—i.e., working out 2 cases $\times 4$ middle matrices is enough for full credit. 21 pts. total; the problem set continues on the next page.)

(5) Draw a picture of the quantum circuit C_G that models a three-node undirected graph G with edges $(1,1),(1,2),(2,3)$. (The graph looks like a lollipop with a stick and a self-loop at the top.) You can draw the quantum circuit by hand or take a screenshot from any of the quantum circuit simulators we've discussed, mindful of any big-endian/little-endian difference.

What we want to do is calculate $|\langle 000 \,|\, C_G \,|\, 000\rangle|^2$, which is the probability that when the circuit is run on input $|000\rangle$, the same value $|000\rangle$ is returned as output. We could write out the 8×8 matrices involved (beginning and ending with the 3-qubit Hadamard transform $\mathbf{H}^{\otimes 3}$) and multiply them, but that would be a horrible amount of work. Tracing the "maze" shown for the triangle graph at the end of the Friday 12/01 lecture could also be gnarly, except that the input and output being $|000\rangle$ removes the need to trace through the tangly forking wires of the Hadamard parts:

- We can immediately place eight positive "mice" at the beginning of the middle section where the wires just go straight across. In linear algebra terms, we have $\mathbf{H}^{\otimes 3}|000\rangle = |{+}{+}{+}\rangle$ = the vector of eight 1s divided by $2\sqrt{2}$.
- At the end of the middle section, the wires going up to $|000\rangle$ at upper right as output will not have any sign flips. Thus the amplitude $\langle 000 \,|\, C_G \,|\, 000\rangle$ will be the sum of the eight "mice" at the end (summing $+1$ for "Phil," -1 for "Anti-Phil"), again divided by $2\sqrt{2}$.

(a) Do the calculation for the above graph G. Again, you need only reproduce the diagrams for the middle sections with the **Z** and **CZ** gates. Say and show which basis states are multiplied by -1 from each gate.

(b) Then change to a new graph G' by moving the loop from node 1 to node 2, so that G' looks like a stick with a loop in the middle. What is the value of $\langle 000 \,|\, C_{G'} \,|\, 000\rangle$ now? (9+9 = 18 pts. total)

(6) Lipton-Regan text, exercises 9.4–9.6 on page 102. (The circuit C is pictured at the bottom of page 42; the four-cycle graph itself is drawn in page 24.) For 9.4, first draw the middle section with sixteen rows—noting e.g. that for the edge $(2,3)$ you will put -1 on the four rows $|0110\rangle, |0111\rangle, |1110\rangle, |1111\rangle$ since those have bits 2 and 3 set to 1—and count how many "mice" change sign. Then for 9.5, say how and why the final sign of each ending mouse depends on how many edges had both nodes with a 1 in the string. Finally, 9.6 is a challenge to see if you can generalize on the pattern you see, either intuitively or via linear algebra as suggested; there are 3 regular-credit points but up to 12 possible extra for convincing linear algebra or logically working when d_G cancels in 9.5. (9+9+3 = 21 pts. total, for 108 regular-credit points on the set, and 18 more possible extra credit points)