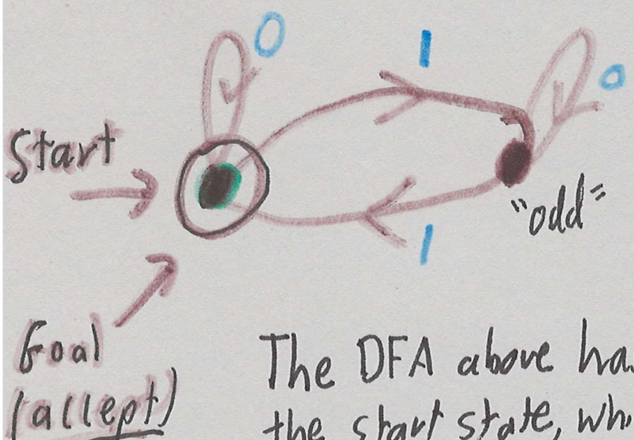
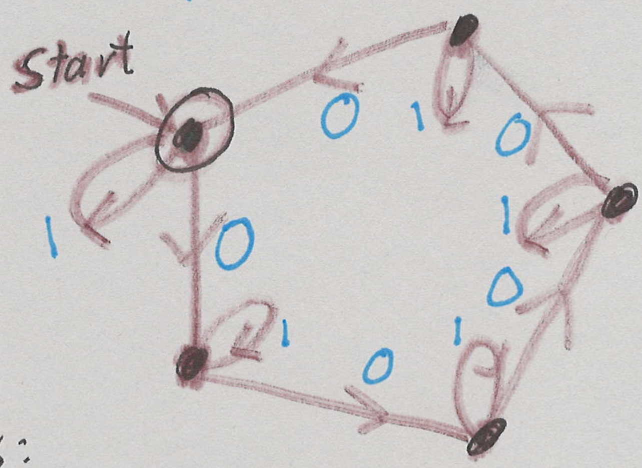
# CSE 491/596 First Lecture Example Fall 202

If not for the needed attention to conferencing logistics, this would have been the final 20 mins. after 30 mins. of overview

One of the great things about computing theory is that there are problems that spring out of elementary definitions and yet continue to challenge the field's top minds. Here is one example using only deterministic finite automata (DFAs). We will define DFAs formally on Wed 9/2 but the idea from pictures will work for now:




The DFA above has two states: the start state, which is also the only accepting state, and the "odd" state, which is a rejecting state. A binary string x is accepted if and only if x has an even number of 1s.

Note: Zero counts as an even number.

This DFA once again makes a single goal state that is the same as the start state. It accepts a binary string x iff the # of 0s in x is a multiple of 5.

Say a DFA M distinguishes x from y if M accepts x and rejects y, or vice-versa. The first DFA distinguishes x = 01100 from y = 01101, for instance. The second DFA does not distinguish these strings, but does distinguish x00 = 0110000 from y00 = 0110100. Here is a surprisingly tricky question:

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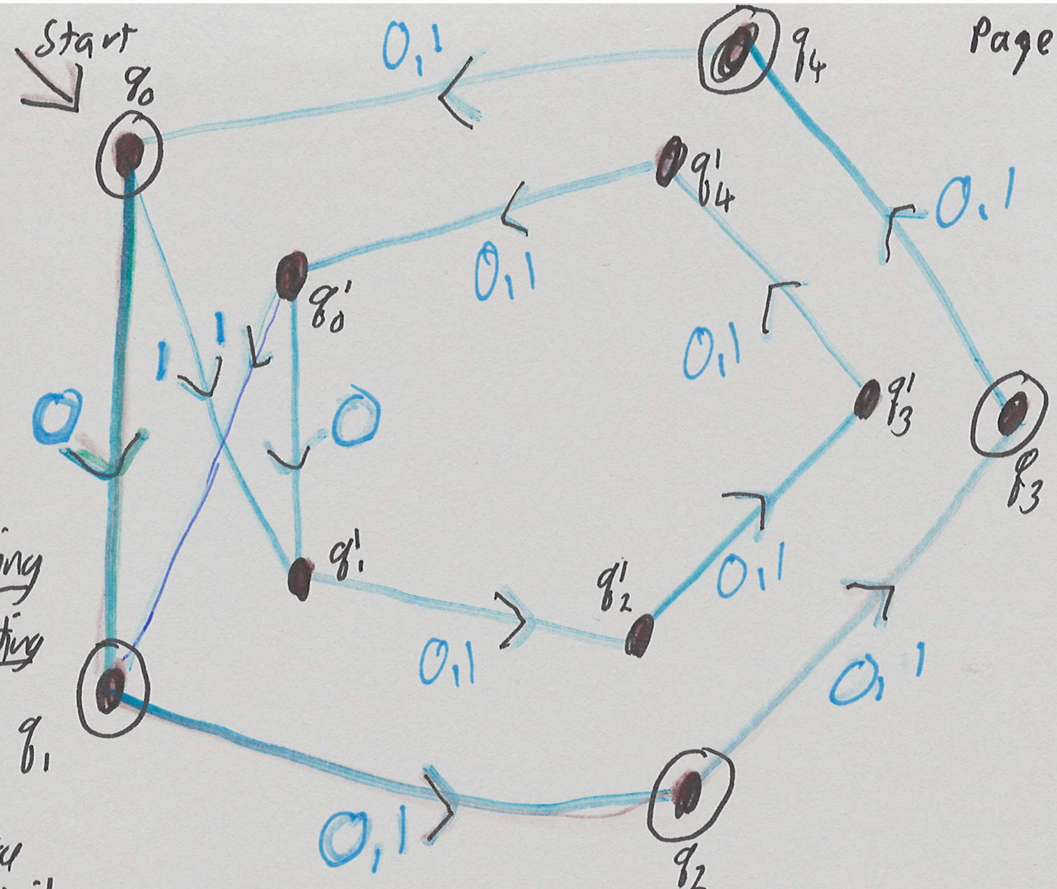
Given any two strings x and y, what is the least # of states in a DFA that distinguishes them?

This DFA combines elements of the first two. I've made the outer ring accepting in place of the inner ring, but for the purpose of distinguishing two strings, complementing the DFA by switching all accepting states to rejecting and vice-versa does not matter. Call it




$\underline{M_{5,0}}$ because its rings have 5 states and its only "crossover" is at $q_0, q_0'$.

Now $M_{5,0}$ does not distinguish $x = 01100$ from $y = 01101$ because only the initial 0 is being read on entering the crossover and it is the same for both strings. If you moved the crossover back to states $q_4$ and $q_4'$, creating what we could call $\underline{M_{5,4}}$, then it would catch the last bits and distinguish the strings. If instead $x' = 010100$ and $y' = 010101$ then $M_{5,0}$ works because the crossover at $q_0, q_0'$ is entered again on the last bit. But for $x'' = 010100$ and $y'' = \underline{1}10101$ the second use of the crossover undoes the immediate use on the initial bit. So $M_{5,0}$ *fails* on $x''$ and $y''$.

Finally, define $f(n)$ = the least $m$ such that *all* different binary $x, y$ of length $n$ can be distinguished by a DFA with at most $m$ states. Getting $m \leq n$ is easy. Can we get $m = O(\log n)$? Nobody knows! DFAs of the form $M_{k,b}$ do get $m = O(\sqrt{n})$.