CSE491/596 Lecture Wed 9/14 Fall 2022

For any relation $R \subseteq A \times B$
define $f_R(a) = \{b : R(a,b)\}$.
$f_R : A \to \mathcal{P}(B)$

Typography Note:

DFA: $\delta : Q \times \Sigma \to Q$

NFA$_\epsilon$: $\delta \subseteq Q \times (\Sigma \cup \{\epsilon\}) \times Q$ or can write

$\hat{\delta} : Q \times (\Sigma \cup \{\epsilon\}) \to \mathcal{P}(Q)$

Macro DFA: $\Delta : \mathbf{Q} \times \Sigma \to \mathbf{Q}$ $\quad \mathbf{Q} \subseteq \mathcal{P}(Q)$

More helpful than $\hat{\delta}$ as given by Sipser and other texts is

$$\underline{\delta} : Q \times \Sigma \to \mathcal{P}(Q)$$

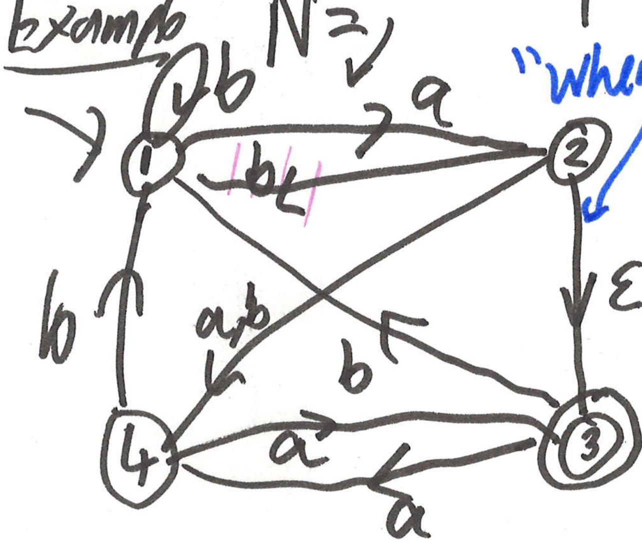
p, q, r: indiv. states

$$\Delta(P,c) = \bigcup_{p \in P} \underline{\delta}(p,c)$$

P, Q, R: sets of states, = "macrostates"

$\mathbf{Q}$: a set of macrostates
= a set of sets of states
= a subset of $\mathcal{P}(Q)$

Example N =



"whenever 2, then also 3"

$\underline{\delta}(1,a) = \{2,3\}$ $\quad \underline{\delta}(1,b) = \{1\}$

$\underline{\delta}(2,a) = \{4\}$ $\quad \underline{\delta}(2,b) = \{4\}$

$\underline{\delta}(3,a) = \{4\}$ $\quad \underline{\delta}(3,b) = \{1\}$

$\underline{\delta}(4,a) = \{3\}$ $\quad \underline{\delta}(4,b) = \{1\}$

$\mathbf{F} = \{R \subseteq Q : 3 \in F\}$ $\qquad \Delta(P,c) = \bigcup_{p \in P} \underline{\delta}(p,c)$

S = just $\{1\}$ since no ϵ out of 1.

$\Delta(\{1\},a) = \delta(1,a) = \{2,3\}$ new $\quad \Delta(\{1\},b) = \delta(1,b) = \{1\}$ not new

$\Delta(\{2,3\},a) = \delta(2,a) \cup \delta(3,a) = \{4\} \cup \{4\} = \{4\}$. new

$\Delta(\{2,3\},b) = \{1,4\} \cup \{1\} = \{1,4\}$ new

The expansion of {3} closes the BFS.

To Discuss: Can we simplify this further?

We know $L(M) = L(N) = ?$ ______

Some Observations:

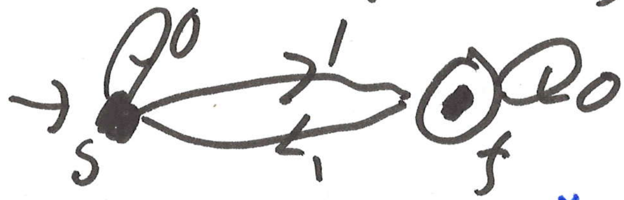
① M processes $x = \underline{ab}$ from S to {1,4}. Hence N can process x from 1 to 1 or from 1 to 4, but not to 2 or 3

② M does not have $\emptyset$ as a state. Thus for all x, N can process x from state 1 to some other state.

Part III: DFA / NFA / (GNFA) $\longleftrightarrow$ Regexp builds on these processing ideas.

$L_{pq} = \{x : M \text{ or } N \text{ can process } x \text{ from state } p \text{ to state } q\}$

Example: Our Parity DFA M =



$L_{ss}^{once} = 0 + 10^*1 \qquad L_{ss} = (L_{ss}^{once})^* = (0 + 10^*1)^*$

$L(M) = L_{sf} = L_{ss} \cdot$ [go from s to f and don't come back to s] $= L_{ss} \cdot \underline{10^*}$

$= (0 + 10^*1)^* 10^*$.