CSE 491/596 Lecture Wed 9/21 Fall 2022

We have proved: If L is a language and S is a PD set for L, and S is infinite, then L is not regular.

Formally, we want to apply the following logic:

<u>If</u> there exists a set $S \subseteq \Sigma^*$ such that S is infinite,

and such that for all x, y in S, $x \neq y$

there exists $z \in \Sigma^*$ s.t. $L(xz) \neq L(yz)$

<u>Then</u> <u>L is not regular</u>.

$\Sigma = \{a, b\}$

<u>Example</u>: $L = \{a^n b^n : n \geq 1\}$. Show L is not regular.

<u>Take</u> $S = \{a^n : n \geq 1\} = a^+$. "Clearly S is infinite."

<u>Let</u> <u>any</u> $x, y \in S$, $(x \neq y)$ <u>be given</u>. Then we can write $x = a^m$, $y = a^n$ for some different $m, n \in \mathbb{N}^+$.

Take $z = b^m$! Then $L(xz) \neq L(yz)$ because:

$xz = a^m b^m$ which is in L, but $yz = a^n b^m$ which is not in L because $m \neq n$. $\therefore$ L is not regular, by <u>MNT</u> $\boxtimes$

Example 2: $L = \{x \in \{a,b\}^* : \#a(x) > \#b(x)\}$

Take $S =$ a^+. "Clearly S is infinite."

Let any $x, y \in S$, $x \neq y$ be given. Then we can helpfully write $x =$ a^m and $y =$ a^n [where wlog. $m < n$]

Take $z =$ b^{n-1}. Then $L(xz) \neq L(yz)$

because $xz = a^m b^{n-1}$ (is legal because $n > m$ so $n \geq 1$ and) is NOT in L since $n-1 \geq m$, whereas $yz = a^n b^{n-1}$ IS in L. ✓

Thus S is PD for L, and since S is infinite, L is non-regular by MN

Example 3: $L = \{x \in \{0,1\}^* : x = x^R\}$ ie. x is a palindrome

Take $S = 0^*1$. Let any $x, y \in S$, $x \neq y$ be given. Then we can write $x = 0^m 1$, $y = 0^n 1$, where $x \neq y$.

Take $z = 0^m$. Then $xz = 0^m 1 0^m \in L$ since it's a palindrome, but $yz = 0^n 1 0^m \notin L$ since $m \neq L$.

$\therefore$ S is PD for L, and since S is infinite, L is non reg. ☒

Proof

Example 4: Say x has a "balancing 1" if $x = u1v$ for some strings u, v with $|u| = |v|$. Show that the language of such x is nonregular