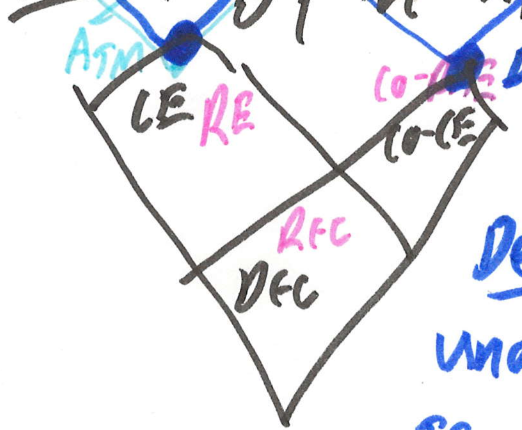
CSE491/596 Lecture Mon 10/17 Fall 2022

Prove $ALL_{TM} = \{\langle M\rangle : L(M) = \Sigma^*\}$ is neither c.e. nor co-c.e.

Proof: By the All or Nothing Switch, $A_{TM} \leq_m ALL_{TM}$



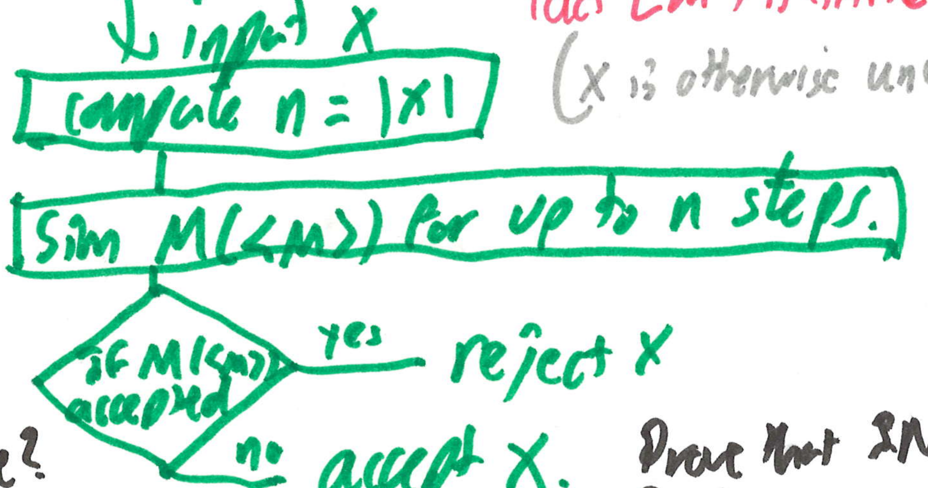
Since A_{TM} is not co-c.e.
ALL_{TM} is not co-c.e. either.

Defn: A language B is *hard* for a class $\mathcal{C}$ under $\leq_m$ if for all $A \in \mathcal{C}$, $A \leq_m B$. If also $B \in \mathcal{C}$, then B is $\mathcal{C}$*-complete*.

To show ALL_{TM} is not c.e. either, show $D_{TM} \leq_m ALL_{TM}$. I.e. map $\langle M\rangle \mapsto M'$ such that

If M does not accept $\langle M\rangle$, then $L(M') = \Sigma^*$
If M does accept its own code, then $L(M') \neq \Sigma^*$; in fact $L(M')$ is finite.

$\langle M\rangle \xrightarrow{f} M' =$



(x is otherwise unused)

Now define the problem
INST: A TM M
QUES: Is $L(M)$ infinite?
Language: $INF = \{\langle M\rangle : L(M) \text{ is infinite}\}$.
Its complement $\{\langle M\rangle : L(M) \text{ is finite}\}$ is called FIN or I_{FIN} because it is an index set.

Prove that INF and I_{FIN} are neither c.e. nor co-c.e.

$A_{TM} \leq_m INF$ via AoNS
$D_{TM} \leq_m INF$ as above!