



The missing text for  $NE_{DFA}$  at top is "INST: Just a DFA  $M$  (no string  $w$ )" and for  $NE_{DTM}$  the missing text is "INST: A deterministic TM  $M$ ".

"Instance Type of the Source Problem"  $A_{TM}$  is "Instance Type of the Target Problem"  $NE_{TM}$  is "Just a Machine"

$\langle M, w \rangle \xrightarrow{f} M'$

A machine on a string. Logical Goal:  $M'$  must obey the requirement of the reduction.

$\langle M, w \rangle \in A_{TM} \iff \langle M' \rangle \in NE_{TM}$

Unpack the meanings:  $M$  accepts  $w \iff L(M') \neq \emptyset$

$\therefore$  We need to show how (given any  $M, w$ ) to build  $M'$  such that  $L(M') \neq \emptyset$  iff  $M$  accepts  $w$ . AOX to diagram  $M'$  as a flowchart.

$M$  and  $w$  can be part of the code of  $M'$ , which we could call  $M'_{\langle M, w \rangle}$

Write the word  $x$

Simulate  $M(w)$  open-endedly. if and when  $M$  accepts  $w$

Accept  $x$ .

This is a computable code mapping, so  $f(M, w)$  is a computable function.  $\odot$  Computability complexity

$\odot$  correctness:

- If  $M$  accepts  $w$ , then  $M'$  accepts every string, so  $L(M')$
- But if  $M$  does not accept  $w$ ,  $M'$  never accepts  $x$ , and is separate, this means  $L(M') = \emptyset$

$\therefore \langle M, w \rangle \in A_{TM} \iff \langle M' \rangle \in NE_{TM}$

Therefore the  $NE_{TM}$  problem

But, the  $NE_{TM}$  language

$NTM N$  st.  $L(N) = L_{NE_{TM}}$

Theorem: For every  $NTM N$  we can build a  $DTM M$  st.  $L(M) = L(N)$ , by using Turing Kit to try all branches of  $N$  and converting that Java code to a  $DTM$ .

The same construction.  $NE_{DTM} = INST: A \text{ DTM } M. QUES = Is L(M) \neq \emptyset?$

also reduces  $A_{TM} \leq_m K_{TM}$  because when  $L(M) = \Sigma^*$ , we will show this undecidable by showing  $A_{TM} \leq_m NE_{TM}$  and using the contrapositive of Monday's theorem.

- If  $M$  accepts  $w$ , then  $M'$  accepts every string, so  $L(M') = \Sigma^* \neq \emptyset$
- But if  $M$  does not accept  $w$ , then  $M'$  never accepts  $x$ , and since  $x$  is separate, this means  $L(M') = \emptyset$ .

$\therefore \langle M, w \rangle \in A_{TM} \iff f(M, w) = \langle M' \rangle \in NE_{TM}$

Therefore the  $NE_{TM}$  problem is undecidable.

But, the  $NE_{TM}$  language is c.e.: We can build an  $NTM N$  st.  $L(N) = L_{NE_{TM}}$ .

Theorem: For every  $NTM N$  we can build a  $DTM M$  st.  $L(M) = L(N)$ , by using Turing Kit to try all branches of  $N$  and converting that Java code to a  $DTM$ .

Guess an  $x \in \Sigma^*$  Run  $M(x)$  If and when it accepts, accept  $\langle M \rangle$ .

Suppose  $A \leq_m B$ . Then:

- If  $A$  is undecidable,  $B$  is undecidable.
- If  $A$  is not c.e., then  $B$  is not c.e.
- If  $A$  is not r.e., then  $B$  is not r.e.