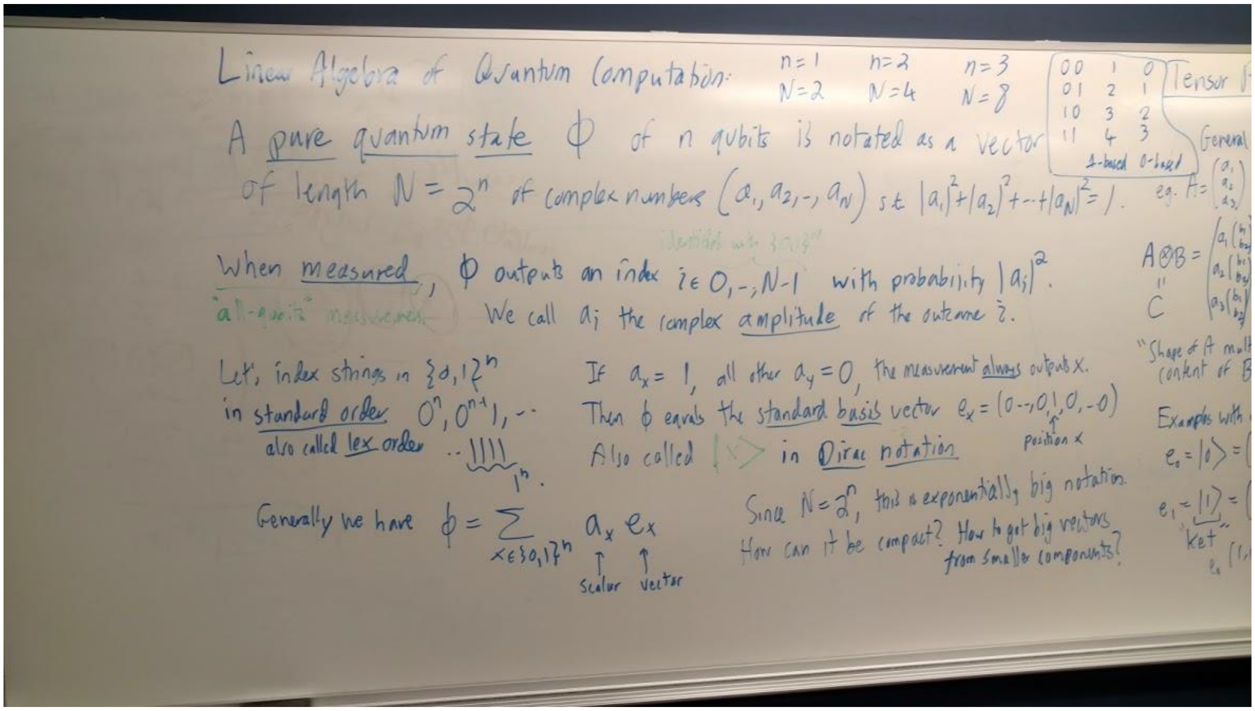
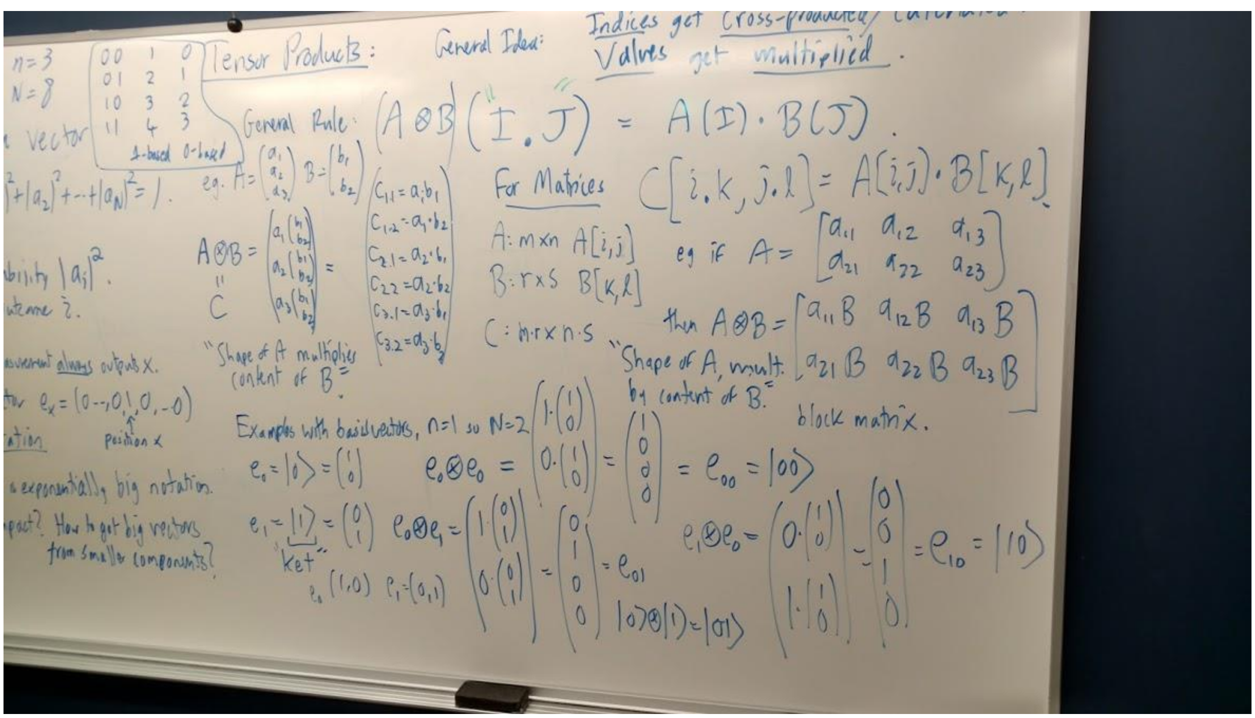
CSE596 Lecture Dec. 2, 2019





Matrix Example $\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \otimes \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$

__Unit vector__ means that $\|\phi\|_2 = \sqrt{\sum_{i=1}^{n} |a_i|^2} = 1$.

__Helpful Rule__: If $A \cdot B$ and $C \cdot D$ are legal matrix products, then $(A \otimes C)(B \otimes D)$ is a legal matrix product and equals $(AB) \otimes (CD)$.

⚠ NOT the same as $\begin{bmatrix} A & 0 \\ 0 & C \end{bmatrix}\begin{bmatrix} B & 0 \\ 0 & D \end{bmatrix} = \begin{bmatrix} AB & 0 \\ 0 & CD \end{bmatrix}$!

if A, B, C, D are all $N \times N$ this is dim $2N \times 2N$ not $N^2 \times N^2$

__Tensor Products__

$A \cdot B$ operation "evolves" separately from the $C \cdot D$ operation. Hence tensor products also convey that the

Similarly, $\phi \otimes \psi$ (phi psi), where ϕ has N outcomes and ψ has R outcomes, conveys the product distribution of $N \cdot R$ pairs of __independent__ outcomes.

__Fact__: Then $\phi \otimes \psi$ is also a unit vector.

__Defn__: A quantum state ϕ is __separable__ if there are smaller ϕ_1, ϕ_2 such that $\phi = \phi_1 \otimes \phi_2$.
Otherwise, ϕ is __entangled__.

__Examples__: $\frac{1}{2}\begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ -1 \end{pmatrix}\frac{1}{\sqrt{2}}$

However $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$ is entangled.

$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \stackrel{?}{=} \begin{pmatrix} a \\ b \end{pmatrix} \otimes \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} ac \\ ad \\ bc \\ bd \end{pmatrix} \therefore \begin{array}{l} ac = 1 \\ ad = 0 \\ bd = 1 \end{array}$

means $a = 0$ or $d = 0$ so $ac = 1$, $bd = 1$ are not both possible

"The Basic Bell Pair" $= \dfrac{|00\rangle + |11\rangle}{\sqrt{2}}$

after __John Bell__. AKA __EPR pair__

$= \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}\frac{1}{\sqrt{2}} = \dfrac{|01\rangle + |10\rangle}{\sqrt{2}}$

General Rule: e.g. $A = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ $B = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$

$A \otimes B = \begin{pmatrix} a_1 \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \\ a_2 \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \\ a_3 \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \end{pmatrix} =$

$\parallel$
C

"Shape of A multiplies content of B".

Examples with basis vectors

$e_0 = |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$e_1 = \underbrace{|1\rangle}_{\text{"ket"}} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $e_0 \otimes$

$e_0\ (1, 0)$ $e_1 = (0,$