

CSE596 Recitation Example, Tuesday Nov. 5

I didn't really answer any questions at the whiteboard. Instead I gave an example of a "SAT-like" reduction, i.e., a reduction to a problem that really just shows (3)SAT dressed up in another form.

$C_1 = (x_1 \vee \bar{x}_2 \vee x_3)$
 $C_2 = (\bar{x}_1 \vee x_4 \vee \bar{x}_5)$

In fact, ALL NFA and its complement are both PSPACE-complete, so not in NP unless PSPACE = NP = coNP.
 Note: $A_{NFA} = \{ \langle N, x \rangle : N \text{ accepts } x \}$ is in P.

In this problem in NP?

Surprising fact: As a function of the number M of states of an NFA N , the smallest string that it does not accept may have size $\approx 2^M - 1$.

① INST: An NFA N
 QUES: Is $L(N) \neq \Sigma^*$?

$\langle N \rangle \in \text{PILL}_{NFA} \Leftrightarrow (\exists x) \langle N, x \rangle \notin A_{NFA}$

② INST: NFA N and a number n (given in unary notation as 0^n)
 QUES: Is there a string x of length n that N doesn't accept?

Now is in NP because $|x| \leq n < |\langle N, 0^n \rangle|$.

And it is NP-complete.

\rightarrow 3SAT & NPC even when every clause has the form $(u \vee v \vee w)$ or $(\bar{u} \vee \bar{v} \vee \bar{w})$ or single \bar{u} or single w .

N_0 has $O(mn)$ states
 Design is easy to build in $O(mn)$ time,
 hence this is a poly-time reduction from 3SAT to A_2 .

Consider the regexp
 $R = 101(0+1)^{n-3}$
 $+ 1(0+1)(0+1)01(0+1)^{n-5}$
 $+ \dots$
 $+ \text{a block for } C_m.$

$3SAT \leq A_2$

$\phi(x_1, \dots, x_n) = C_1 \wedge \dots \wedge C_m$

Idea: N_0 will accept an $x \in \{0,1\}^n$ if there is some clause that x does not satisfy.

Then R has no Kleene $*$'s and $L(R) = \{0,1\}^n \Leftrightarrow \phi \notin SAT$.

Then a yes answer to A_2 will happen $\Leftrightarrow (\exists x \in \{0,1\}^n) N_0$ does not accept x
 \Leftrightarrow there is no clause that x does not satisfy.
 $\Leftrightarrow x$ satisfies every clause $\Leftrightarrow \phi \in SAT$.

Say for example $C_1 = (x_1 \vee \bar{x}_2 \vee x_3)$
 $C_2 = (\bar{x}_1 \vee x_4 \vee \bar{x}_5)$

INST: An NFA N
 QUES: Is $L(N) \neq \Sigma^*$?

$\langle N \rangle \in \text{PILL}_{NFA} \Leftrightarrow$

INST: NFA N and A_2 instance size is x
 QUES: Is there a string x that N doesn't accept?

Now is in NP

And it is NP-complete.

\rightarrow 3SAT & NPC even when every clause has the form $(u \vee v \vee w)$ or $(\bar{u} \vee \bar{v} \vee \bar{w})$ or single \bar{u} or single w .