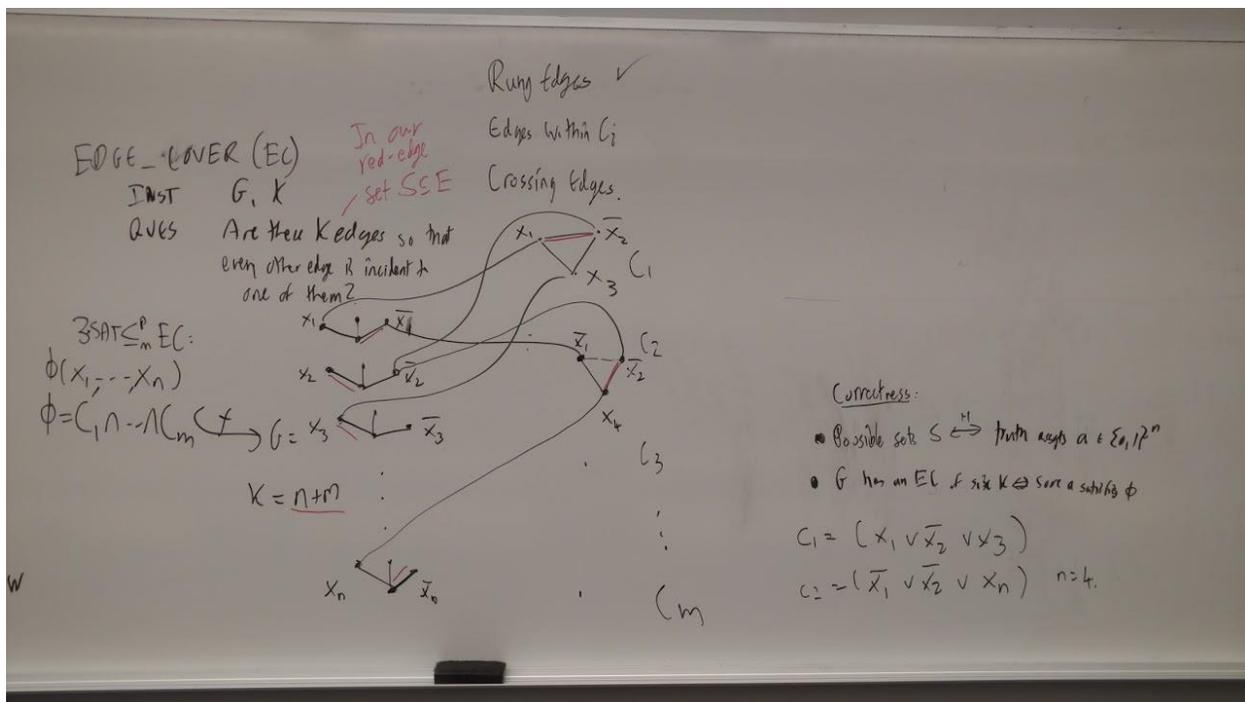
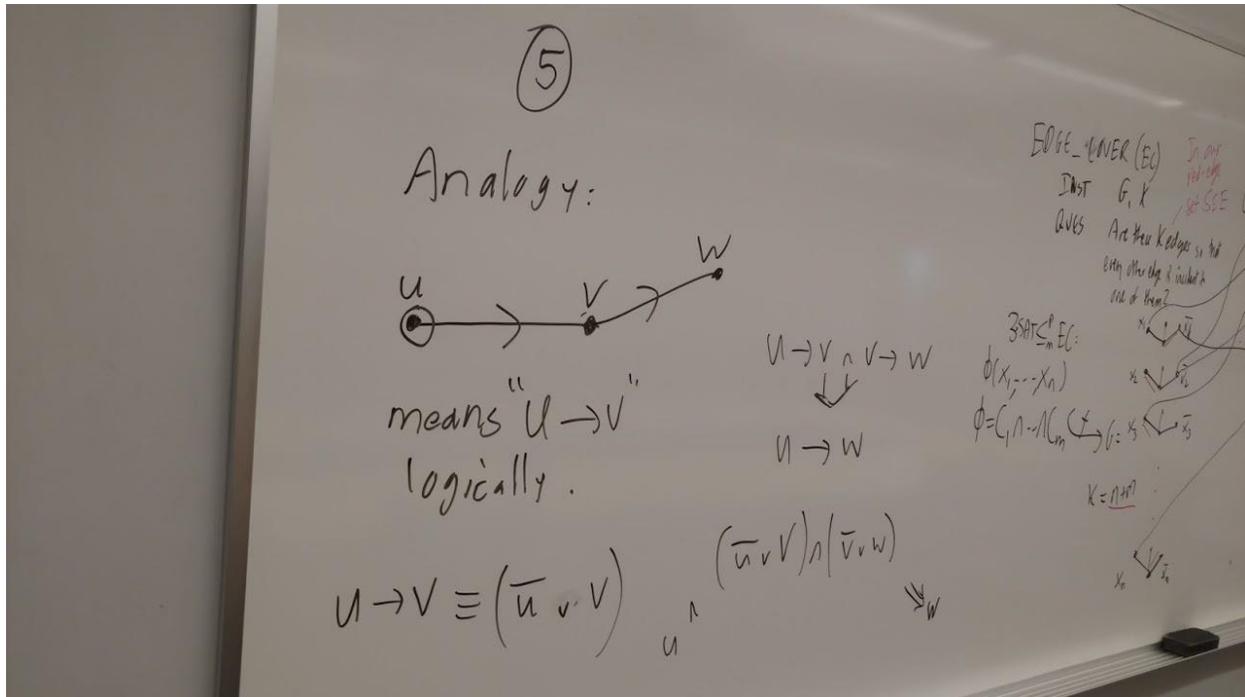


Problems 5 and 1 were the main questions. I did an example of using the "rungs and clause gadgets" architecture to reduce 3SAT to another an NP-complete problem.



The single picture doesn't show how I unfolded the logic of the reduction by first designing the rungs to offer a choice between the left edge for setting  $x_i = \text{true}$  or the right-hand edge for  $x_i = \text{false}$ .

Here was a followup question aimed at getting reductions in the right direction:

Given any graph  $G = (V, E)$ , define its line graph  $\overline{G} = (E, E')$  where

$$(e, f) \in E' \iff e \text{ and } f \text{ are incident in } G.$$

Q: Does the mapping  $G \xrightarrow{f} \overline{G}$  do

$$\text{Dom\_Set} \leq_m^P \text{EC} \quad \text{or} \quad \text{EC} \leq_m^P \text{Dom\_Set?}$$

Correctness:

The answer is the latter, because this  $f$  makes edges (black) **become** vertices (red). The last bit was a fact relevant to problem 4: if two classes have different *closure properties*, then we know they cannot be equal, even though we may not be able to prove that neither contains the other.

We don't know  $NP \subset DLBA$  •  $\{A : A \leq_m^P B \text{ for some } B \in NP\} = NP$  "NP is closed (downward) under  $\leq_m^P$ "  
 We haven't ruled out  $DLBA \subset NP$  But •  $\{A : A \leq_m^P B \text{ for some } B \in PSPACE\}$   
 But we do know  $DLBA \neq NP!$

(5)  $\{A : A \leq_m^P TQBF\}$  **EDGE-LOWER (EC)**  $G, K$  In our red-edge set SSE  
 $= PSPACE$  since PSPACE is closed under  $\leq_m^P$  INST  $G, K$   
 $\neq DLBA$  by the Space Hierarchy Theorem. QUES Are there Kedges so that every other edge is incident to one of them?

Analogy:  
  
 means "u → v" logically.  
 $u \rightarrow v \wedge v \rightarrow w$   
 $\Downarrow$   
 $u \rightarrow w$

$3SAT \leq_m^P EC:$   
 $\phi(x_1, \dots, x_n)$   
 $\phi = C_1 \wedge \dots \wedge C_m \xrightarrow{f} G = x_3$   
 $K = n+m$