CSE191, Fall 2013 — Final Exam Makeup — May 18-19, 2015

Open notes but not text or Internet access, closed neighbors, 170 minutes. Please do ALL TEN problems in the exam booklets provided. Please *show all your work*; this may help for partial credit. The exam totals 240 pts., subdivided as shown.

The difference of sets A and B can be written either $A - B$ as in the text, or $A \setminus B$ as in lectures. The symmetric difference is $A \oplus B$ as in the text; although in other sources you may have seen the notation $A \Delta B$ for it.

(1) ($5 \times 6 = 30$ pts. total)

Note that $P \longleftrightarrow Q$ and $P \oplus Q$ each have 2 true and 2 false cases, whereas $P \wedge Q$, $P \vee Q$, $P \longrightarrow Q$, and P NAND Q (among some others) are split 3–1: either one true and 3 false cases, or one false and 3 true cases. And the trivial functions giving always-true and always-false are split 4–0. Call propositions of the $\longleftrightarrow$ and $\oplus$ and trivial kind "even," and call the others "odd."

Each of the following assertions has two propositional variables and/or predicates (or one of each) expressed in prose, possibly negated. For each one, write it more symbolically, *eliminating negations if you can*, and classify it as "even" or "odd." For example, "Anyone who can't stand the heat must get out of the kitchen" can be rewritten using the contrapositive as

$(\forall x)$: InKitchen$(x) \longrightarrow$ CanStandHeat(x),

and is odd. If you add the clause, "and anyone who can stand the heat must stay," then it becomes an equivalence, which is even. The assertions are:

(a) Don't both eat ice cream and drink whiskey.

(b) $P \vee Q$ and $(\neg Q) \vee (\neg P)$.

(c) A function $f : X \longrightarrow Y$ fails to be 1–1 exactly when its reversal is not a function.

(d) He can run but he can't hide.

(e) If the quarterback can't stay in the pocket, the team won't win.

Note: You may perceive multiple ways to interpret one of these, but before asking a question, please see if the two ways give the same even/odd answer. In any event, *please show your reasoning in the exam booklet(s) provided.*

(2) (12 + 27 = 39 pts.)

Consider the following five premises:

1. A naive person does not understand schemers.
2. All persons who share their leads on anyone are naive.
3. All good detectives understand schemers.
4. Only a good detective could have caught Moriarty.
5. Sherlock Holmes caught Moriarty, after another detective shared his leads.

In addition, here are some premises which the original author of a similar exercise took for granted. They are written in a formal style which suggests how you can formalize the above:

- $(\forall X)$: $\texttt{detective}(X) \longrightarrow \texttt{person}(X)$.
- $(\forall X, Y, M)$: $\texttt{caught}(X, M) \wedge$ $\texttt{sharedLeadsOn}(Y, M) \longrightarrow Y \neq X$.

First write the above five numbered premises symbolically. Then for each of the following propositions, say whether it follows from the above premises. If so, give a proof using symbolic logic rules. If not, give a reason why not.

(a) Sherlock Holmes was naive

(b) Some other detective was naive.

(c) Sherlock Holmes was not naive.

(3) (24 pts. total)

For each of the following assertions about sets A, B, C, say whether it is always true. If so, give a truth table for the corresponding logical formula; if not, give a Venn diagram where it is false. Recall that $\oplus$ stands for symmetric difference.

(a) $A \oplus B \subseteq A \cup B$.

(b) $C \subseteq A \wedge C \subseteq B \longrightarrow C \subseteq A \oplus B$.

(c) $A \cup (B \cap C) = (A \cup B) \cap C$.

(4) ($4 \times 3 \times 2$ = 24 pts.)

For each of the following functions of real numbers, and for each of the properties of (i) being defined on all of $\mathbb{R}$, (ii) being 1–1 on its domain, and (iii) being onto $\mathbb{R}$, say whether the property is

- True,
- Almost True—that is, true except for 1 or 2 real numbers, or
- Really False—that is, false and not "almost true."

You should have a grid of answers with 5 rows and 3 columns, and entries "True," "Almost True," or "Really False." Here $|x|$ means the absolute value of the number x.

(a) $f(x) = 3x$.

(b) $f(x) = |x|/x^2$.

(c) $f(x) = \ln x$.

(d) $f(x) = \frac{x-3}{2x-3}$.

(5) (18 pts.)

Use the Euclidean algorithm to prove that the integers 28 and 45 are relatively prime. Then work backwards from its output to find integers s and t such that $28s + 45t = 1$.

(6) (24 pts. total)

How many different ways are there to choose two integers, each of absolute value at most n, such that either both numbers are negative or both are non-negative? It does not matter which integer comes first. Is the number of ways a perfect square? Prove your answer.

(7) (24 pts.)

Consider the recurrence relation $r(n) = 3r(n-1) + 2^{n-1}$ with initial conditions $r(0) = 0$, $r(1) = 1$. Prove by induction that for all n, $r(n) = 3^n - 2^n$.

(8) (24 pts. total)

Let V be the power set of $\{a, b, c\}$. Define a directed graph $G = (V, E)$ by drawing an edge from a subset A of $\{a, b, c\}$ to a subset B if there is an element $i \in B$ such that $A = B \setminus \{i\}$. To get you started, the node for $\emptyset$ has three out-arrows, to the nodes for $\{a\}$, $\{b\}$, and $\{c\}$.

(a) Draw the graph. For labels on the vertices, even better than giving the subsets, give the corresponding binary bit-string representations.

(b) What does the in-degree of each node equal?

(c) Now consider the undirected graph $G' = (V, E')$ made by ignoring the arrows on the edges. Describe it, including saying what the degrees of the nodes are now.

(d) Does the directed graph G have any cycles? How about the undirected graph G'? Is G' connected?

(e) In G', show that for every node A there is a node A' such that the neighborhoods of A and A' do not overlap. Put another way, every path between A and A' has length at least 3. Can you say what A' "is" in terms of A?

(9) (15 pts. total)

This time define $V = \{2, 3, 4, 5, 6\}$. Define a graph $G = (V, E)$ by putting (i, j) into E if and only if neither i nor j divides the other evenly.

(a) Give an adjacency matrix for G. Is G directed or undirected? You are welcome to draw G.

(b) Is there any node whose neighborhood is all of G?

(c) Show that G is not bipartite, by finding a 3-cycle in G (that is, a triangle).

(d) If you delete node 5 and all the edges into it, is the resulting graph connected?

(10) (6 × 3 = 24 pts.) *True/False:* No justifications are asked for, though they could help for partial credit. Please write `true` or `false` in full.

(a) For all sets A and B, the symmetric difference $A \oplus B$ equals A if and only if $B = A$.

(b) There is no bijection between $\mathbb{N}^+$ and $\mathbb{N}$.

(c) It is technically false but essentially true that Cartesian product of sets is associative.

(d) If $f : \mathbb{R} \longrightarrow \mathbb{R}$ and $g : \mathbb{R} \longrightarrow \mathbb{R}$ are 1–1 functions, then the sum function $h(x) = f(x) + g(x)$ is 1–1.

(e) If $f : \mathbb{R} \longrightarrow \mathbb{R}$ and $g : \mathbb{R} \longrightarrow \mathbb{R}$ are onto $\mathbb{R}$, then the sum function $h(x) = f(x) + g(x)$ is onto $\mathbb{R}$.

(f) With all quantifiers over the same domain, if $(\exists x)P(x)$ is true and $(\exists x)Q(x)$ is true, then $(\exists x)[P(x) \wedge Q(x)]$ is true.

End of Exam