

Reading: As stated before, skim Chapter 3, and especially ignore the proof given about the Halting Problem. The one fact related to Chapter 3 that I will lean on is that if $p(x)$ is a polynomial of degree d with leading term ax^d , and $q(x)$ is a polynomial of degree d with leading term bx^d , then not only are they “Theta-Of” each other, but we get the stronger statement that as x goes to infinity, their ratio $p(x)/q(x)$ converges to a/b . The upshot of this is that when comparing polynomials for their growth rates, only the degree and the leading coefficient matter—all the lower-order terms do *not* matter. An example of this alluded to in lecture is that $\sum_{k=0}^n k^r$, which the text gives the exact formula for on page 166 for $k = 1, 2, 3$, is approximately $n^{r+1}/(r+1)$. You can get this approximation by pretending the sum is really an integral—here’s where it helps to start the sum from 0 not 1—and doing it. The basic fact is that the integral approximation of a sum is not only “good up to Theta,” in fact its ratio to the exact sum approaches 1. That is, it’s like the case where a and b are equal, both equal to $1/(r+1)$.

Hence read Chapter 4, sections 4.1–4.3 for next week. These and induction in Chapter 5 will be the other major topics before the second prelim exam, which is likely to be held on Friday Nov. 22.

(1) Let B^A stand for the set of functions from A into B (not necessarily with range all of B). Show that if $B \subseteq D$, then $B^A \subseteq D^A$. How about if $A \subseteq C$ instead—is $B^A \subseteq B^C$ —how would you vote? (6 pts.)

(2) Rosen, back on page 126, problem 38. (9 pts.)

(3) Rosen, page 137, problem 40. If you understand the point about the relation to addition modulo 2, this is not a “star” exercise, so only 9 pts.

(4) Rosen, page 137, problem 50, (a) and (b) only. Show your work, i.e., give reasoning as well to make a “college answer.” (6+6 = 12 pts.)

(5) Rosen, page 153, problem 22(a,d), but in place of (a) do $f(x) = (4 - 3x)/(3 - 2x)$. If you say it is not a bijection because it is not a function, can you “patch” it by excluding one or more isolated bad points from the domain? If you say it (or your “patch” of it) is a function but not a bijection, say which of being 1-1 or onto fails, or both. If you say it is not onto, can you “patch” it further by excluding one or more isolated bad points from the range? If your “patch” then becomes a bijection, prove it by showing how given any y in the patched range, you can uniquely solve the equation you get for x . (24 pts. total)

(6) Rosen, page 168, problem 16, (a,b,c) only. (6+6+6 = 18 pts.)

(7) Rosen, page 169, problem 32, (a,c) only (6+6 = 12 pts., for 90 on the set).