Closed book, closed-notes except for 1 sheet, closed neighbors, 48 minutes. Please do ALL FOUR problems on the separate exam sheet provided. Please *show all your work*; this may help for partial credit. The exam totals 80 pts., subdivided as shown.

(1) ($7 \times 3 = 21$ pts.)
For each of the following assertions, either:

- write an equivalent proposition in the form "if P then Q";
- write an equivalent proposition in the form "P if and only if Q"; or
- say that it cannot be put into either form.

Note that P and Q can be negations of propositions, but they will not have to be any more "compound" than that. It is fine to write your answers symbolically or in prose. For example, "We can go outside only if it isn't raining" can be answered as "GoOutside $\longrightarrow \neg$ Raining" or as "if we go outside, then it is not raining." There are no predicates, and it is OK to regard the constants `true` and `false` as disallowed.

(a) To enter Mexico you must have a passport.

(b) The government will stay shut down, unless and until Congress votes the money to fund it.

(c) I've got music and I've got rhythm.

(d) Put your hands up or I'll shoot.

(e) You cannot have your cake and eat it too.

(f) Either you and I stand, or you and I fall.

(g) The cat is alive and the cat is not alive.

(2) (18 pts. total)
For each of the following, say whether or not it is a tautology. If not, give one truth assignment that refutes it; if so, give a proof using equivalences such as De Morgan's laws that transforms it into an obviously always-true statement.

(a) $A \longrightarrow (B \longrightarrow A)$.

(b) $A \vee (B \wedge C) \longrightarrow A \wedge (B \vee C)$.

(c) $A \wedge (B \vee C) \longrightarrow A \vee (B \wedge C)$.

[Exam continues overleaf.]

(3) (6 + 5 + 12 = 23 pts.)

First, write the following two assertions using quantifiers and the predicates `rotten(X)`, `spoiled(X)`, and `inDenmark(X)`.

(a) Everything that is spoiled becomes rotten.

(z) Something is rotten in the state of Denmark.

(The second is a line from Shakespeare's play *Hamlet*; you may ignore the words "the state of" when translating.)

Second, rewrite your answer for (a) to be the more specific statement

(a′) Everything that is spoiled in Denmark becomes rotten.

Third, prove (z) given (a) or (a′) (your choice) plus the facts (b) `spoiled(hamlet)` and (c) `inDenmark(hamlet)`. Please label lines of your proof with further letters and state the rule you used in each step. (It is OK to use the abbreviations UI for universal instantiation, UG for universal generalization, EI for existential instantiation, and EG for existential generalization. It is kind-of OK to take shortcuts with the rules called simplification and conjunction in the text, which lectures called "the houseekeeping rules for AND.")

(4) (18 pts. total)

For each of the following quantified statements, say whether it is true over each of the following domains.

(i) The integers, $\mathbb{Z}$.

(ii) The positive integers, $\mathbb{N}^+$.

(iii) The real numbers, $\mathbb{R}$.

(iv) The positive real numbers, $\mathbb{R}^+$ (excluding zero too).

The statements are:

(a) $(\forall x)(\exists y)\ y < x$.

(b) $(\forall x)(\exists! y)\ y = x^2$.

(c) $(\forall x)(\exists! y)\ x = y^2$.

Recall $\exists!$ is the uniqueness quantifier—so (b) is equivalent to $(\forall x, z)(\exists y)[y = x^2 \wedge (z = x^2 \longrightarrow z = y)]$. Please make a 3×4 labeled table of your answers, or write them in the form "(a)-(v) false," etc. Whenever you say one is false, give a counterexample value of x—the points are 1 for each table entry and 6 total for counterexamples.

End of Exam