CSE396, Spr'10 Prelim I Mar. 3, 2010

Open book, open notes, closed neighbors, 75 minutes. Do ALL FIVE problems in the exam booklet(s) provided. Note the choice in problem (3). Please *show all your work* in those booklets—this may help for partial credit. The exam totals 100 pts., subdivided as shown.

Notation: *All* problems on this exam use alphabet $\Sigma = \{ a, b \}$. For all strings w and chars c, $\#c(w)$ stands for the number of occurrences of c in w.

(1) (5 × 3 = 15 pts.) *True/False.*

Please write out the words `true` and/or `false` in full. No justifications are needed. *Be sure to write your answers in the exam books.*

(a) It is possible to have regular languages A and B such that $A \cap B$ is not regular.

(b) Every subset of a regular language is a regular language.

(c) For all languages A (even non-regular ones), $A \cdot A^* \cup \{ \epsilon \} = A^*$.

(d) If N is an NFA with k states, then there is always a DFA M with at most $3k$ states such that $L(M) = L(N)$.

(e) For any two states p, q in a DFA M, there is a regular expression α that matches precisely those strings that M can process from state p to q—i.e., such that $L(\alpha) = L_{pq}$.

(2) (15 + 15 = 30 pts.)

Consider the following NFA N. (It has $Q = \{ 1, 2, 3 \}$, start state $s = 1$, $F = \{ 1 \}$ making the start state the only final state, and arcs $\delta = \{ (1, a, 1), (1, b, 3), (2, a, 1), (2, b, 2), (2, \epsilon, 3), (3, a, 2), (3, b, 1) \}$.)

(a) Convert N into a DFA M such that $L(M) = L(N)$.

(b) Write a regular expression for $L(N)$.

(3) (6 + 12 = 18 pts.)

Let A be the language of strings x over alphabet $\{ a, b \}$ such that:

- x begins with 'a' and ends with 'a'
- between every two 'a's in x there are at least two 'b's.

For example, the string $abba$ belongs to A, but $ababa$ does not.

(a) For each of the following strings, say whether it belongs to A:

(i) ϵ (ii) a (iii) $abbaa$

(b) YOUR CHOICE: *Either* write a regular expression α such that $L(\alpha) = A$, *xor* write an NFA N such that $L(N) = A$. (A DFA where you don't bother showing a dead state and arcs to it counts as an NFA.)

(4) (16 pts.)

For each finite automaton M on the left, find a regular expression α on the right such that $L(M) = L(\alpha)$. Note that the FAs are labeled (a)–(d) and the expressions (i)–(v), with one regular expression left over that is not used. *Good exam practice:* write out the regular expression as well as the Roman numeral next to the letter (a)–(d) of the FA you think it goes with.

(i) $(ab + ba)^*$

(ii) $a^*(bab)^*$

(iii) $a^*b(ab)^*$

(iv) $(a + ba^*b)^*$

(v) $(a + bab)^*$

(5) (6 + 15 = 21 pts.)

Define $L = \{ xby : x, y \in \{ a, b \}^*, \ \#a(x) = \#b(y) \}$.

(a) For each of the following strings, say whether it belongs to L:

(i) ϵ (ii) $abbb$ (iii) bbb

(b) Prove by express use of the Myhill-Nerode technique that L is not a regular language.

END OF EXAM.