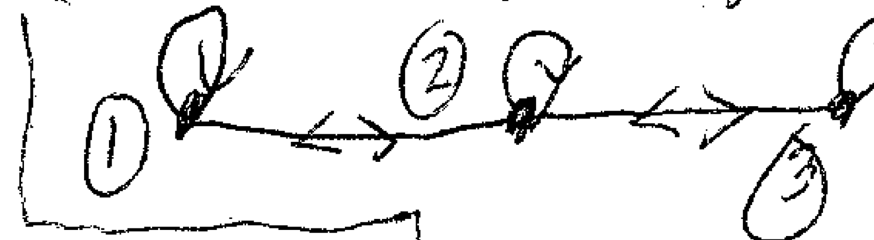
CSE 396 Problem Set 1 Answer Key Spring 2015

(1)

(a) Reflexive and symmetric but not transitive: {(1,1), (1,2), (2,1), (2,2), (2,3), (3,2), (3,3)}

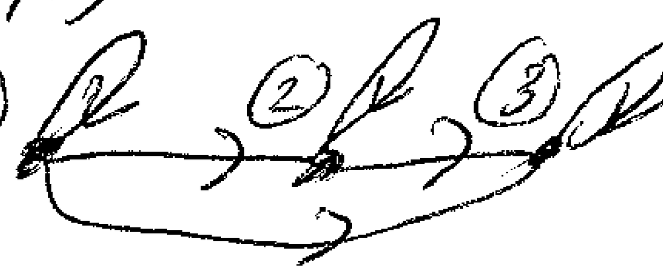
(b) Reflexive and transitive but not symmetric: {(1,1), (1,2), (2,2)} No (2,1)



(1,2) & (2,3) but no (1,3).

Or: $|a-b| \le 1$ for integers a, b.

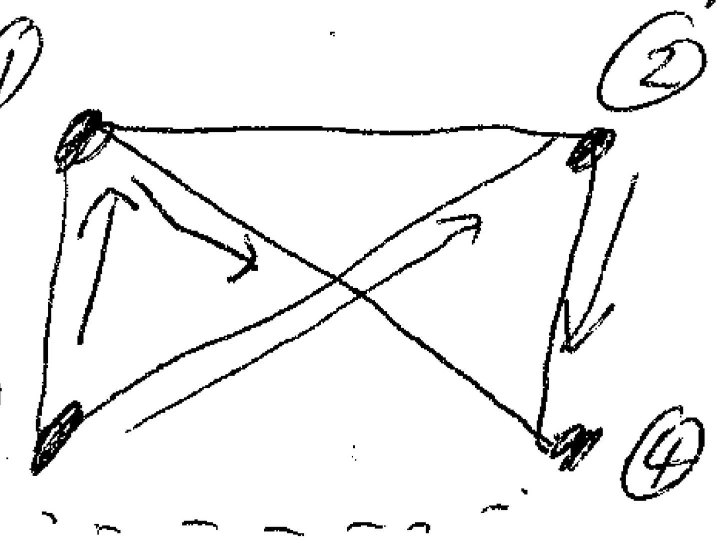
Or: {(1,1), (1,2), (2,2), (1,3), (2,3), (3,3)}



Or: $a \le b$ for integers a, b.

(c) Symmetric and transitive but not reflexive: The most 'Zen' answer is: The empty relation on a non-empty set. A little less Zen: $U = \{1,2\}$, $R = \{(1,1)\}$. Not reflexive since $(2,2) \notin R$.

(d) Graph is



(e) deg(1) = 3
deg(2) = 3
deg(3) = 2
deg(4) = 2

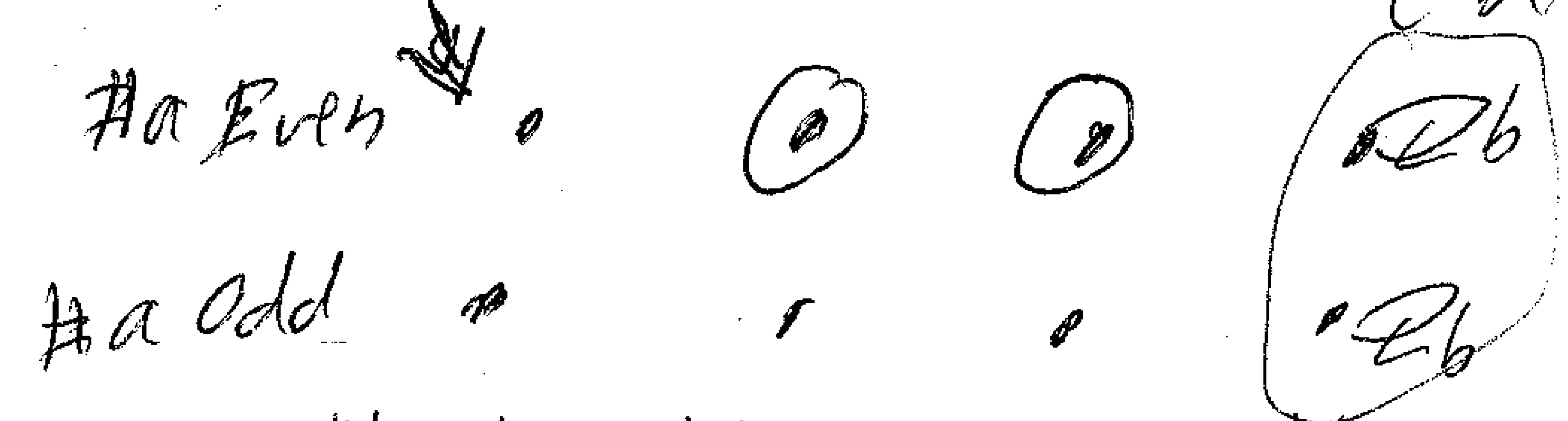
(f) Two paths of length 2 are $3 \to 1 \to 4$ and $3 \to 2 \to 4$, shown by arrows in the diagram.

(g) Not transitive. Easiest reason is $(3,2) \in E \wedge (2,4) \in E$ but $(3,4) \notin E$.

(h) Does adding the undirected edge (3,4) make the graph transitive? Technically no. In the definition "$xRy \wedge yRz \to xRz$," it has to hold even when $z = x$. So you'd still fail since $(3,4) \in E \wedge (4,3) \in E$ but $(3,3) \notin E$. Hence G needs self-loops too. [BUT, partly in view of some people disallowing self-edges in undirected graphs, many people in my field speak as if clique graphs are transitive.]

(2) $\boxed{\text{1.4c in text:}}$ $L = \{x \in \{a,b\}^* : \#a(x) \text{ is even and } (\#b(x) = 1 \text{ or } \#b(x) = 2)\}$

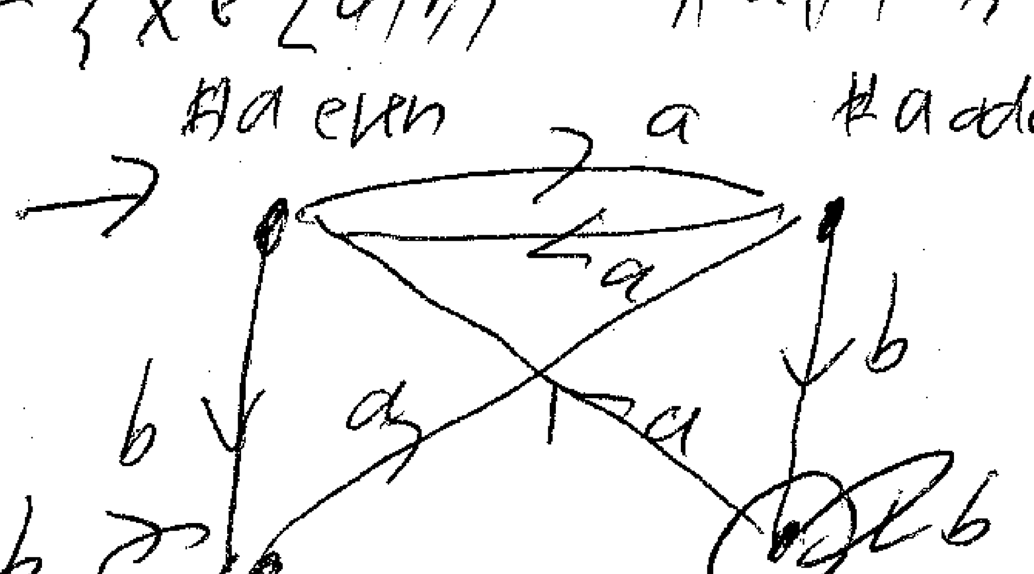
The layout has a natural grid structure:



Once you have the design grid, with comments and with accepting states circled, the rest is clear: Arcs on a go a⟲a in each column. Arcs on b go right in each row, except for loop when #b>2. The two states at right can be one dead state.

$\boxed{\text{1.4f in text}}$

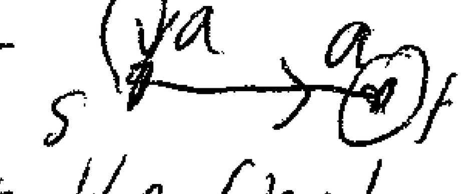
$L = \{x \in \{a,b\}^* : \#a(x) \text{ is odd } \& \ x[|x|-1] = 'b'$.



Last char = a (or start)

Last char = b

But, if we have an NFA like N =



$L(N) = aa^* = a^+$, complementing the final states does not work: with $F' = \{s\}$, the NFA N' now accepts all of a^*.

Bonus: 1.14 in text. To complement a DFA $M = (Q, \Sigma, \delta, s, F)$, build $M' = (Q, \Sigma, \delta, s, F')$ with $F' = Q \setminus F$. Then $L(M') = \overline{L(M)}$.