**CSE396** **Problem Set 3 Answer Key** **Spring 2021**

(1) Using *TopHat*, the "Worksheet" titled *S21 HW3 Online Part.* There are 11 questions—not 10—but still summing to 20 points total. *Answers given there.*

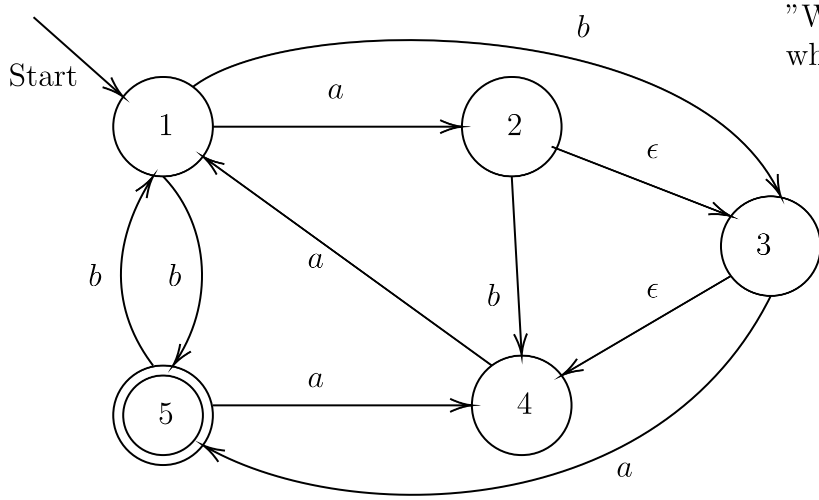
As a non-graded warmup for the written part, try converting SHIPS to WHALE going through English words by changing one letter at a time. For a cryptic hint, to get it in as few as five steps, you may need to have a little more than your wits about you.

*Answer:* Shortest way I know is SHIPS-WHIPS-WHITS-WHITE-WHILE-WHALE. Usually we only say "not one whit" but "whits" is a legal word to pluralize.

(2) Convert the following NFA $N$ into an equivalent DFA (24 pts.). In symbols, $N = (Q, \Sigma, \delta, s, F)$ with $Q = \{1, 2, 3, 4, 5\}$, $\Sigma = \{a, b\}$, $s = 1$, $F = \{5\}$, and

$$\delta = \{(1, a, 2), (1, b, 3), (1, b, 5), (2, \epsilon, 3), (2, b, 4), (3, \epsilon, 4), (3, a, 5), (4, a, 1), (5, a, 4), (5, b, 1)\}.$$

Show the steps of the conversion process clearly. Then, for a further 18 pts., answer the questions after the following picture of the NFA:




"Whenever 2 then 3 and 4; and whenever 3 then 4 (not nec. 2)."

$S = \{1\}$ only
$F =$ "any set with 5"

| $\delta$ | $a$ | $b$ |
|---|---|---|
| 1 | $\{2, 3, 4\}$ | $\{3, 4, 5\}$ |
| 2 | $\emptyset$ | $\{4\}$ |
| 3 | $\{5\}$ | $\emptyset$ |
| 4 | $\{1\}$ | $\emptyset$ |
| 5 | $\{4\}$ | $\{1\}$ |

(a) The picture looks a little like a boat, but the main worry is that since $2^5 = 32$, the DFA could need a boatload of states. The two $\epsilon$-arcs, however, already rule out (at least) half the possible states. Explain why and which sixteen states are never possible in a DFA that comes from a 5-state NFA with those two $\epsilon$-arcs. (6 pts.)

(b) Does your $M$ have a dead state? If you say yes, give a shortest string $x$ that reaches it from start—i.e., so that $N$ cannot process $x$ from state 1. (3 pts.)

(c) Does your $M$ have the "omni" state $\{1, 2, 3, 4, 5\}$? If you say yes, give a shortest string $y$ that reaches it—i.e., so that $N$ can process $y$ from its start state to any of its five states. (3 pts.)

(d) Does your $M$ have an accepting state $P$ so that once a string $u$ enters $P$, there is no way to get to a rejecting state, so that $uv \in L(M)$ for all $v \in \Sigma^*$? (3 pts.)

(e) Is there a simple way you can combine two states into one and get an equivalent machine? (3 pts., with the general understanding that your answer should include some reasoning, not just a bare "yes" or "no" here, giving 42 total on the problem)

*Answers:*

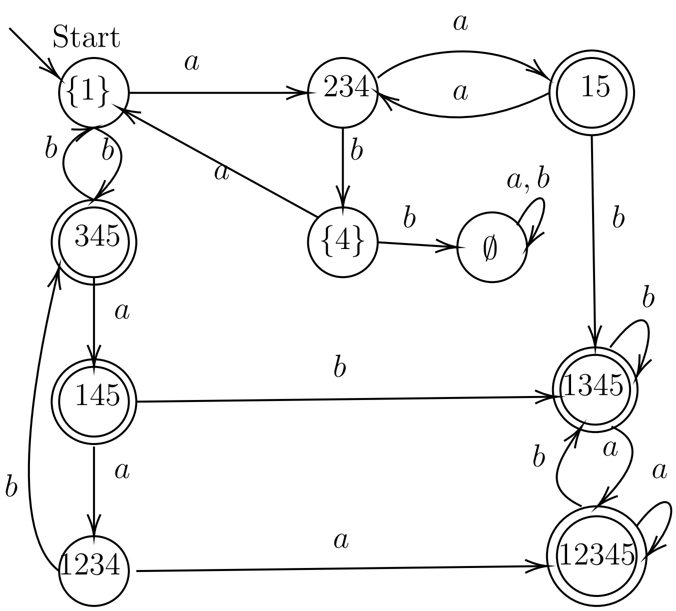
(a) Among 2,3,4, the $\epsilon$-arcs knock out the possibilities $\{2\}$, $\{3\}$, $\{2,3\}$, and $\{2,4\}$. Each is impossible with-or-without 1 and/or 5, which multiplies each case by 4. So this makes $4 \times 4 = 16$ impossible set-states. In full, the impossible ones are $\{2\}$, $\{1,2\}$, $\{2,5\}$, $\{1,2,5\}$, $\{3\}$, $\{1,3\}$, $\{3,5\}$, $\{1,3,5\}$, $\{2,3\}$, $\{1,2,3\}$, $\{2,3,5\}$, $\{1,2,3,5\}$, $\{2,4\}$, $\{1,2,4\}$, $\{2,4,5\}$, and$\{1,2,4,5\}$. The others are: $\emptyset$, $\{1\}$, $\{5\}$, $\{1,5\}$, $\{4\}$, $\{1,4\}$, $\{4,5\}$, $\{1,4,5\}$, $\{3,4\}$, $\{1,3,4\}$, $\{3,4,5\}$, $\{1,3,4,5\}$,$\{2,3,4\}$, $\{1,2,3,4\}$, $\{2,3,4,5\}$, and $\{1,2,3,4,5\}$. Not all of them actually come up, however, in the breadth-first search.

The table for $\underline{\delta}$ has been added to the above diagram. The BFS begins with $S = \{1\}$:

$$\Delta(\{1\},a) = \{2,3,4\}, \quad \Delta(\{1\},b) = \{3,4,5\} \qquad \text{From first row of table.}$$

Two new states. Once you get practice, this can be done "by sight" directly into a table as well (new states shown in magenta), or even right when drawing the DFA:

| $\Delta$ | $a$ | $b$ |
|---|---|---|
| $\{1\}$ | $\{2,3,4\}$ | $\{3,4,5\}$ |
| $\{2,3,4\}$ | $\{1,5\}$ | $\{4\}$ |
| $\{3,4,5\}$ | $\{1,4,5\}$ | $\{1\}$ |
| $\{1,5\}$ | $\{2,3,4\}$ | $\{1,3,4,5\}$ |
| $\{4\}$ | $\{1\}$ | $\emptyset$ |
| $\{1,4,5\}$ | $\{1,2,3,4\}$ | $\{1,3,4,5\}$ |
| $\{1,3,4,5\}$ | $\{1,2,3,4,5\}$ | $\{1,3,4,5\}$ |
| $\emptyset$ | $\emptyset$ | $\emptyset$ |
| $\{1,2,3,4\}$ | $\{1,2,3,4,5\}$ | $\{3,4,5\}$ |
| $\{1,2,3,4,5\}$ | $\{1,2,3,4,5\}$ | $\{1,3,4,5\}$ |




(b) The DFA has a dead state, and the (unique) shortest string reaching it is *abb*.

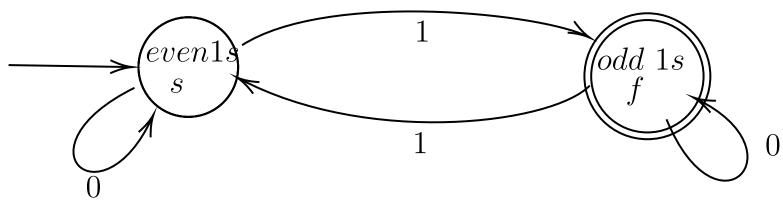
(c) The DFA has the "omni" state $\{1,2,3,4,5\}$, and there are several strings of shortest length 4 that reach it: *aaba*, *baaa*, and *baba*.

(d) The "omni" state can go to state $\{1,3,4,5\}$ on a $b$, but that state can only go back to "omni" or to itself. So once a string hits either one, "eternal acceptance"is in force. Two strings $u$ that are even shorter than the strings reaching "omni" are *aab* and *bab*.

(e) The two eternally-accepting states can be condensed into one such state that loops only to itself, without changing the language of the machine. Thus this is also an example where the BFS procedure does not crank out a minimum-size DFA.

(3) Using $\Sigma = \{0,1\}$ this time, write a regular expression for the language of binary strings in which the number of 1's is odd. First write an expression from scratch. Then compare it with two of the expressions for $L_{s,f}$ in the two-state DFA for this language (with the start state $s$ meaning "even" and $f$ meaning "odd" regarding the current count of 1's). Write a few general words with your opinion of how easy it is to tell that they are equivalent. (Yes, this is "shooting the breeze," but that is an important skill for navigating this material in connection with verbal thinking, so worth 12 pts., for 74 total on the set.)

*Answer:* For "even number of 1's" we might come up with $(0^*10^*10^*)^*$. That would make "odd number of 1's" become $(0^*10^*10^*)^*\,10^*$. Or something like that. Now from the machine:




We have $L_{s,s} = (0 + 10^*1)^*$. This already looks different from the "even number of 1's" expression above. By the symmetry, $L_{f,f}$ is the same. So the expressions we get for $L_{s,f}$ from the lecture formulas are $L_{s,s} \cdot 10^* = (0 + 10^*1)^*\,10^*$ and $0^*1 \cdot L_{f,f} = 0^*1\,(0 + 10^*1)^*$.