Name and St.ID#:____

CSE396, Spr'16 — Prelim I Makeup — Mar. 9, 2016

Closed books and laptops, one notes sheet allowed, closed neighbors, 75 minutes. Do ALL FIVE problems (note (4) is short) **on these exam sheets**. Extra sheet(s) may be requested later. Please *show all your work*—this may help for partial credit. The exam totals 100 pts., subdivided as shown.

Notation: All problems on this exam use alphabet $\Sigma = \{ a, b \}$, and $\#c(x)$ stands for the number of occurrences of the character c in the string x. The symmetric difference of two languages A and B is defined by $A \triangle B = (A \setminus B) \cup (B \setminus A)$, where $\setminus$ is difference of sets.

(1) (21 pts.)

Let us regard any string x over the alphabet $\Sigma = \{a, b\}$ as being uniquely composed of alternating *blocks* of a's and b's. For instance, the string $x = aababbbaa$ has five blocks: aa, b, a, bbb, and aa in that order. The empty string has zero blocks; strings in $a^* \cup b^*$ are composed of one block; every other string has at least two blocks.

Let $A = \{x \in \{ a, b \}^* : \text{every block in } x \text{ has odd length}\}$. Design a deterministic finite automaton (DFA) M such that $L(M) = A$. A node-arc diagram that shows the start and final states clearly is good enough—you need not write out tables or "$M = (Q, \Sigma, \delta, s, F) \ldots$" etc. You must provide a prose *comment* on the meaning of each state for full credit.

(2) (18 + 18 = 36 pts.)

Let N be the NFA defined by $N = (Q, \Sigma, \delta, s, F)$ with $Q = \{1, 2, 3\}$, start state $s = 1$, $F = \{1\}$ making the start state the only final state, and arcs $\delta = \{(1, a, 1), (1, b, 3), (2, a, 1), (2, b, 2), (2, \epsilon, 3), (3, a, 2), (3, b, 1)\}$.

(a) Calculate a DFA M such that $L(M) = L(N)$ (no "comments" needed if the method is clear).

(b) Calculate a regular expression R such that $L(N) = L(R)$.

(3) (5 × 3 = 15 pts.) *True/False.*

Please write out the words `true` and/or `false` **in full**. No justifications are needed. *Be sure to write your answers in the exam books.*

(a) For all regular expressions α and β, $\alpha^*(\beta\alpha)^* = (\alpha\beta)^*\alpha^*$.

(b) If A and B are regular languages, then so is $A \cdot B \setminus (A \cap B)$.

(c) When an NFA N is converted into a DFA M such that $L(M) = L(N)$, the DFA M must always have a dead state.

(d) If A is an infinite regular expression, then every regular expression α such that $L(\alpha) = A$ must have a Kleene-* in it somewhere.

(e) If A and B are languages and $A \cdot B$ contains a string of length 5, then $A \cup B$ must contain a string of even length.

`(a) ________   (b) ________   (c) ________   (d) ________   (e) ________`

(4) (only 7 pts.)

Suppose $A \subseteq \Sigma^*$ is a regular language, and suppose there is a string $y \in \Sigma^*$ such that for all $z \in \Sigma^*$, $yz \notin A$. Let M be the unique DFA with the least number of states such that $L(M) = A$. Must M have a dead state? Justify your yes/no answer briefly.

(5) (21 pts.)

Over $\Sigma = \{a, b\}$, define $L = \{\, vaw : \#a(v) = \#b(w) \,\}$. Prove via the Myhill-Nerode technique that L is not a regular language.