CSE396 Prelim IIm Spring 2016

Closed book, closed notes (except for 1 sheet), closed neighbors, 75 minutes. Do ALL FOUR problems *in the exam booklets provided*. Problem (4) has a *choice*—you must do (a) and EXACTLY ONE of the options (b) or (c). *Show all your work in the exam booklets*—this may help for partial credit. The exam totals 100 pts., subdivided as shown.

Notation: As in lectures, $\#a(x)$ stands for the number of a's in the string x, etc.

(1) (5 × 3 = 15 pts.) *True/False.*

Please write out the words `true` and/or `false` in full. No justifications are needed. *Be sure to write your answers in the exam books.*

(a) The union of two context-free languages is always a context-free language.

(b) There is a regular language L whose complement $\tilde{L}$ is not a context-free language.

(c) Let $G = (V, \Sigma, \mathcal{R}, S)$ be a context-free grammar with the rule $S \longrightarrow SS$. If a given string x is ambiguous in G, then so is xx.

(d) Some context-free languages cannot be accepted by any deterministic Turing machine.

(e) Every context-free grammar in Chomsky normal form is unambiguous.

(2) (24 pts.)

Let $E = \{\, x \in \{\, a, b \,\}^* : \#b(x) - \#a(x) \text{ is a multiple of } 3 \,\}$. Let G be the context-free grammar

$$S \longrightarrow aA \mid SbB \mid AB, \qquad A \longrightarrow aa \mid b \mid aB, \qquad B \longrightarrow SaS \mid AA.$$

Prove that $L(G) \subseteq E$. (Reasonable proof shortcuts are OK.)

(3) (31 pts.)

Let G be the context-free grammar

$$S \longrightarrow AbS \mid BaS \mid A \mid B \mid \epsilon, \qquad A \longrightarrow aA \mid a, \qquad B \longrightarrow bB \mid b.$$

(a) Prove that $L(G) = \Sigma^*$, where $\Sigma = \{\, a, b \,\}$. (Hint: Given $x \in \Sigma^*$, $x \neq \epsilon$, you may (or may not) need 3 or 4 cases, depending on how you word things. You may use separate "lemmas" for the languages of strings derived by A and B without involving them in the overall induction, and some other reasonable proof shortcuts are OK. 24 pts.)

(b) Is G ambiguous? If you say yes, find an ambiguous string $x \in L(G)$ and give two different parse trees for x; if you say no, prove your answer. (7 pts.)

(4) (30 pts.)

Define $L = \{\, a^i b^j c^k : i = j + k,\ i, j, k \geq 0 \,\}$. Do (a) and EXACTLY ONE of (b) or (c). In whichever case you choose, you must write some prose comments that explain how your G, M_1, or M_2 works—and you need not prove correctness formally.

(a) Design a context-free grammar G such that $L(G) = L$. Then modify your G into a context-free grammar G' such that $L(G') = L \setminus \{ \epsilon \}$ and G' has no ϵ-rules.

(b) Design a single-tape deterministic Turing machine M_1 such that $L(M_1) = L$. OR,

(c) Design a deterministic pushdown automaton M_2, coded as a two-tape deterministic Turing machine that obeys the pushdown restrictions, such that $L(M_2) = L$.

END OF EXAM.