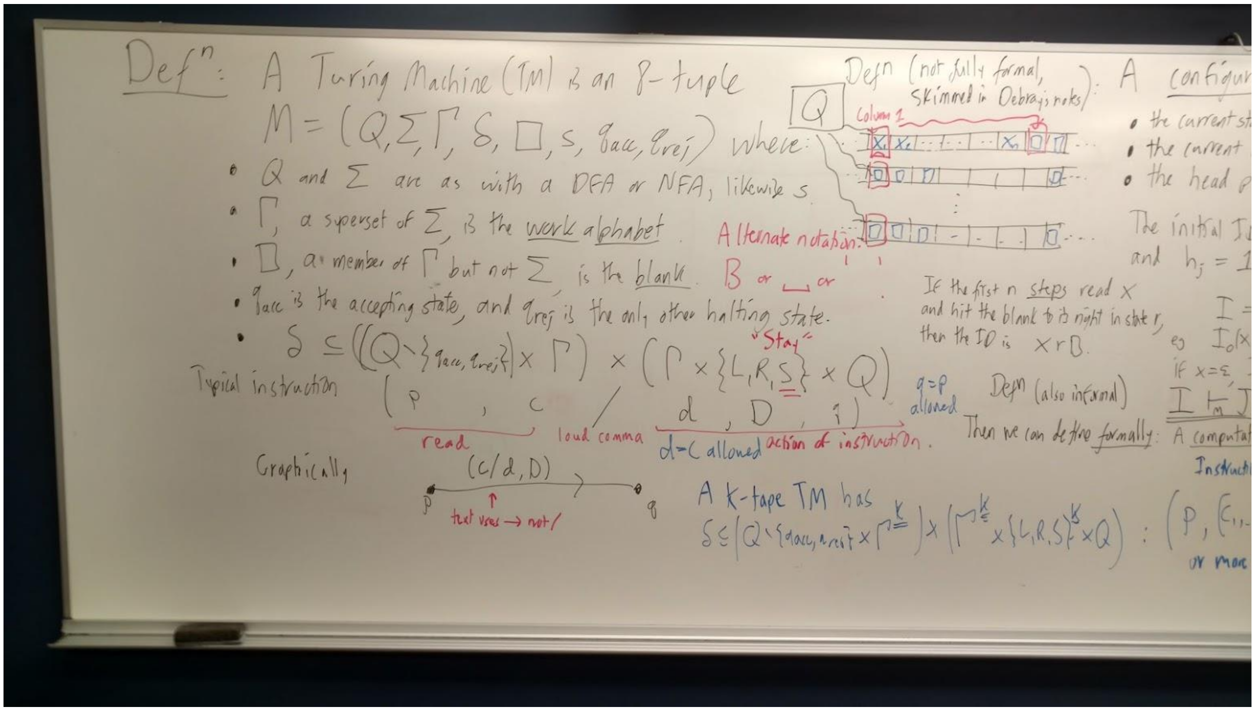
CSE596 Lecture Wed. 9/18



Def^n: A Turing Machine (TM) is an 8-tuple
$M = (Q, \Sigma, \Gamma, \delta, \Box, s, q_{acc}, q_{rej})$ where:

- Q and Σ are as with a DFA or NFA, likewise s.
- Γ, a superset of Σ, is the work alphabet.
- $\Box$, a member of Γ but not Σ, is the blank. Alternate notation: B or ␣ or
- q_{acc} is the accepting state, and q_{rej} is the only other halting state.
- $\delta \subseteq ((Q \setminus \{q_{acc}, q_{rej}\}) \times \Gamma) \times (\Gamma \times \{L, R, \underline{\underline{S}}\} \times Q)$ "Stay"

Typical instruction $(\underbrace{p \quad , \quad c}_{\text{read}} \;/\; \underbrace{d \;,\; D \;,\; q}_{\text{action of instruction}})$ loud comma; $d=c$ allowed; $q=p$ allowed

Graphically $p \xrightarrow{(c/d,\,D)} q$ ↑ text uses → not /

Defn (not fully formal, skimmed in Debray's notes):

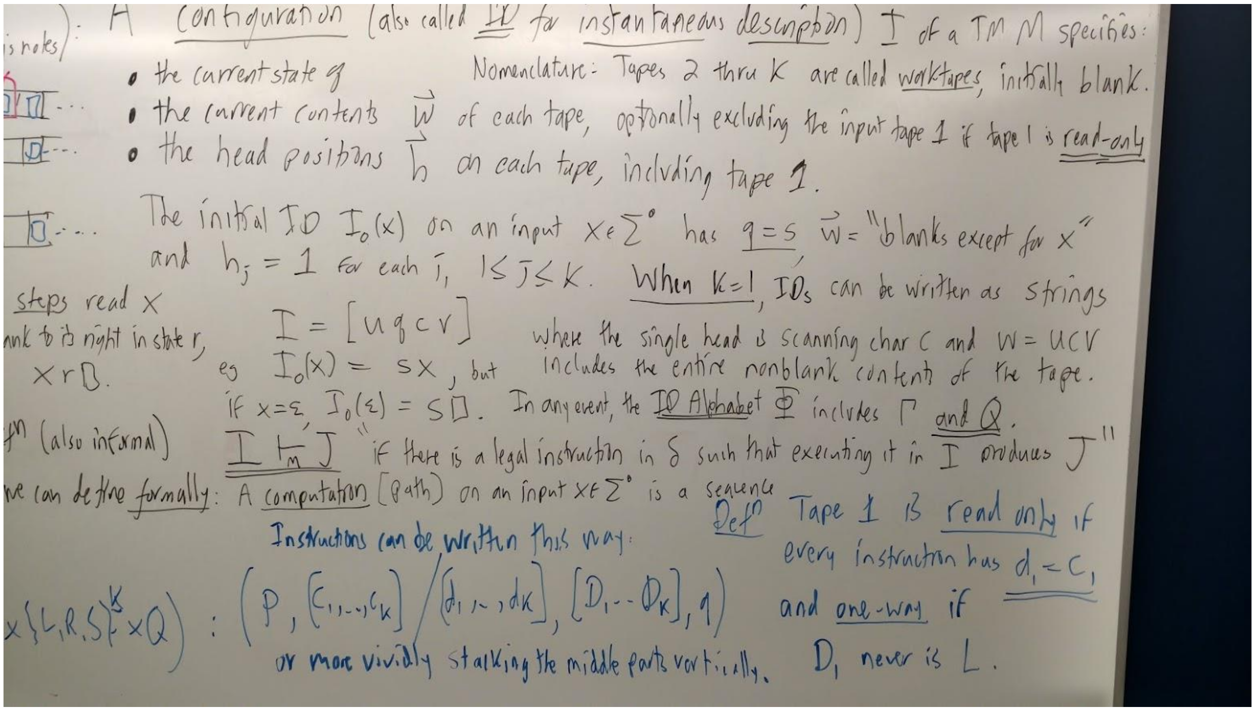
Q
Column 1
x_1 x_2 … x_n $\Box$ $\Box$
$\Box$ $\Box$ $\Box$ … $\Box$
$\Box$ $\Box$ $\Box$ … $\Box$

If the first n steps read x and hit the blank to its right in state r, then the ID is $x r \Box$.

Defn (also informal)

Then we can define formally:

A k-tape TM has
$\delta \subseteq (Q \setminus \{q_{acc}, q_{rej}\} \times \Gamma^{\underline{\underline{k}}}) \times (\Gamma^{k} \times \{L, R, S\}^{k} \times Q)$:



A configuration (also called ID for instantaneous description) I of a TM M specifies:

- the current state q
- the current contents $\vec{w}$ of each tape, optionally excluding the input tape 1 if tape 1 is read-only
- the head positions $\vec{h}$ on each tape, including tape 1.

Nomenclature: Tapes 2 thru k are called worktapes, initially blank.

The initial ID $I_0(x)$ on an input $x \in \Sigma^*$ has $q = s$, $\vec{w}$ = "blanks except for x" and $h_j = 1$ for each j, $1 \leq j \leq k$. When k=1, IDs can be written as strings

$I = [u\,q\,c\,v]$ where the single head is scanning char c and $w = ucv$ includes the entire nonblank content of the tape.

eg $I_0(x) = sx$, but if $x = \epsilon$, $I_0(\epsilon) = s\Box$. In any event, the ID Alphabet Φ includes Γ and Q.

$\underline{I \vdash_M J}$ "if there is a legal instruction in δ such that executing it in I produces J"

A computation (path) on an input $x \in \Sigma^*$ is a sequence

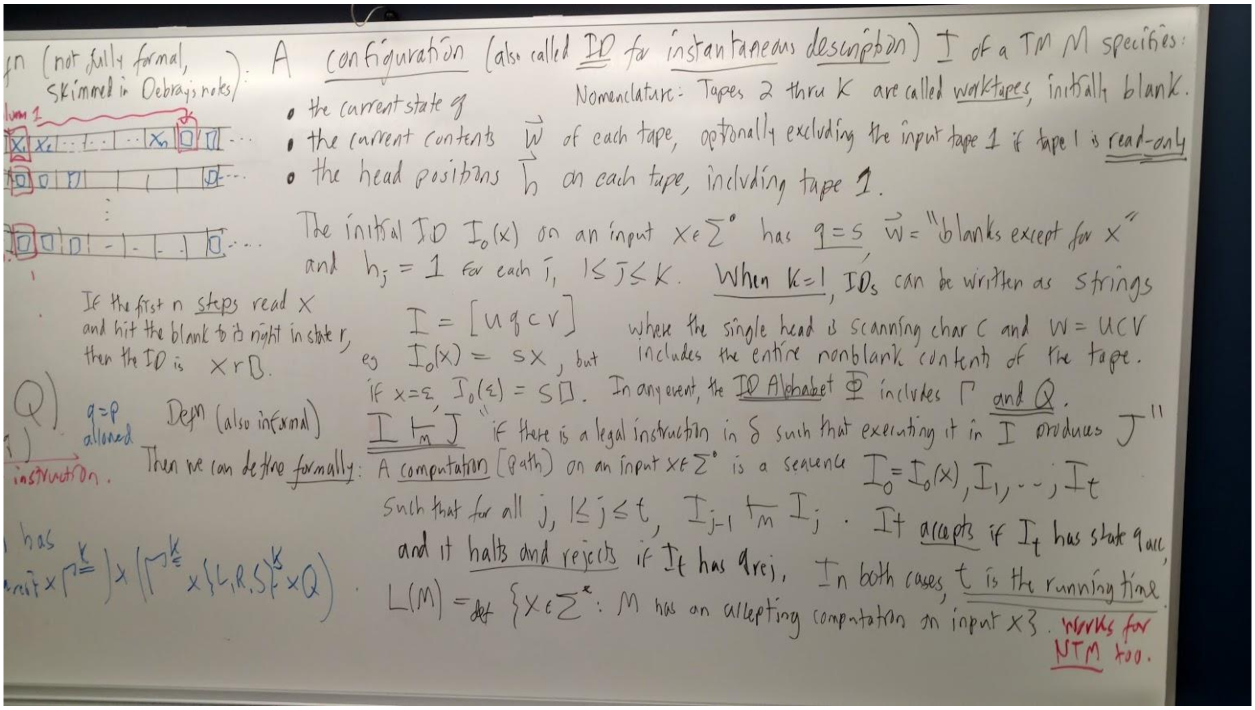
Instructions can be written this way:

$(p, [c_1, \dots, c_k] / [d_1, \dots, d_k], [D_1 \dots D_k], q)$

or more vividly stacking the middle parts vertically.

Def^n Tape 1 is read only if every instruction has $d_1 = c_1$ and one-way if D_1 never is L.

Next pic overleaf has the same after overwriting the blue at the bottom.



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If the first n steps read x and hit the blank to its right in state r, then the ID is $x r \Box$.

Q)

$q = p$ allowed

Defn (also informal) "$I \vdash_M J$" if there is a legal instruction in δ such that executing it in I produces J"

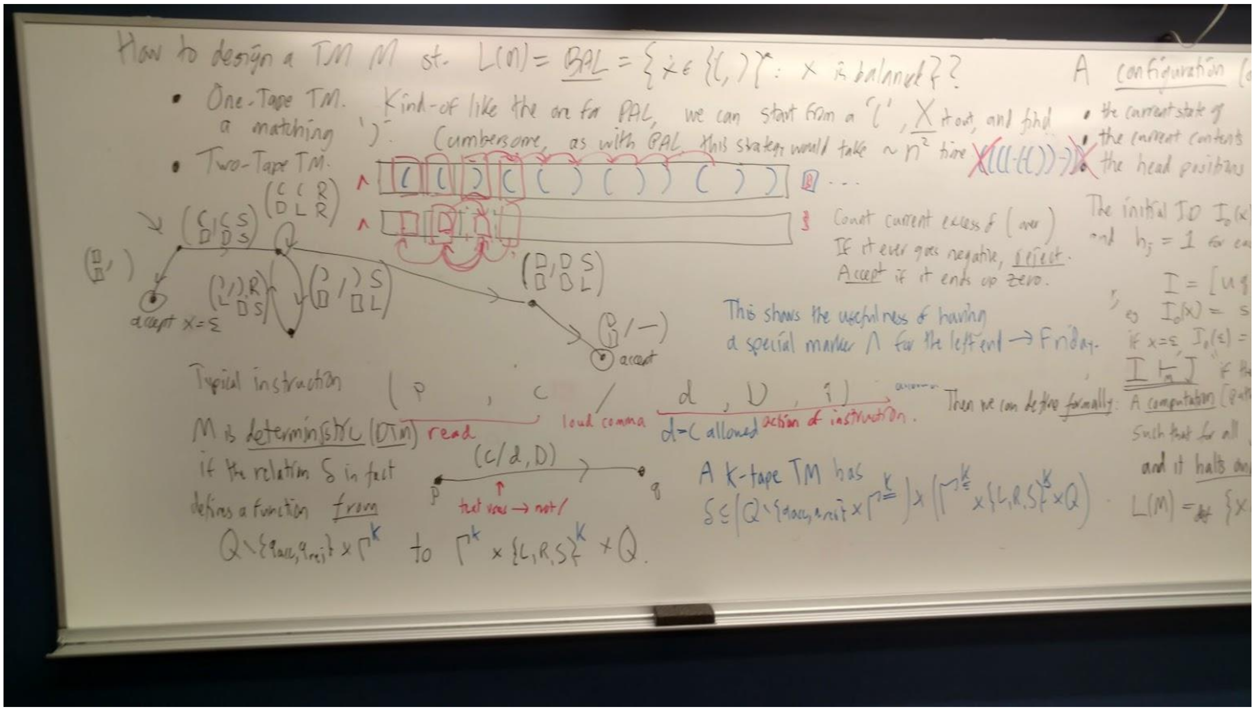
instruction.

Then we can define formally: A computation (path) on an input $x \in \Sigma^*$ is a sequence $I_0 = I_0(x), I_1, \dots, I_t$ such that for all j, $1 \le j \le t$, $I_{j-1} \vdash_M I_j$. It accepts if I_t has state q_{acc}, and it halts and rejects if I_t has q_{rej}. In both cases, t is the running time.

...has

$\dots_{rej} \times \Gamma^k) \times (\Gamma^k \times \{L,R,S\}^k \times Q)$.

$L(M) =_{def} \{x \in \Sigma^* : M$ has an accepting computation on input $x\}$. Works for NTM too.



How to design a TM M s.t. $L(M) = BAL = \{x \in \{(,)\}^* : x$ is balanced$\}$?

- One-Tape TM. Kind-of like the one for PAL, we can start from a '(', X it out, and find a matching ')'. Cumbersome, as with PAL. This strategy would take $\sim n^2$ time ~~(((-(())-))~~
- Two-Tape TM.

∧ (( ) (( ) ( )) ( )) □ ...
∧ □ □
$\binom{(,(\ S}{\Box\ \Box\ S}$
$\binom{(\ (\ R}{\Box\ L\ R}$
$\binom{\Box}{\Box},)$
accept $x = \varepsilon$
$\binom{)\,/\,)\ R}{L\ \ \Box\ S}$
$\binom{)\,/\,)\ S}{\Box\ \ \Box\ L}$
$\binom{\Box,\Box\ S}{\Box\ \ \Box\ L}$
$(\wedge/-)$
accept

Count current excess of (over). If it ever goes negative, reject. Accept if it ends up zero.

This shows the usefulness of having a special marker ∧ for the left end → Friday.

Typical instruction $(p, c / d, D, q)$

read — loud comma — action of instruction. $d = c$ allowed

M is deterministic (DTM) if the relation δ in fact defines a function from $Q \setminus \{q_{acc}, q_{rej}\} \times \Gamma^k$ to $\Gamma^k \times \{L,R,S\}^k \times Q$.

A k-tape TM has $\delta \subseteq (Q \setminus \{q_{acc}, q_{rej}\} \times \Gamma^k) \times (\Gamma^k \times \{L,R,S\}^k \times Q)$.

Then we can define formally:

A configuration (...
- the current state q
- the current contents
- the head positions
The initial ID $I_0(x)$ ... and $h_j = 1$ for ea...
$I = [u q$ ...
eg $I_0(x) = s$ ...
if $x = \varepsilon$, $I_0(\varepsilon) =$ ...
$I \vdash_M J$ if ...
A computation (path ...
Such that for all ...
and it halts and ...
$L(M) =_{def} \{x$ ...

Friday’s lecture will include a demo of the completed balanced-parens TM.