April 24, 2015

Professor Gary G. Yen
IEEE CIS Dissertation Awards Committee Chair
Oklahoma State University School of Electrical and Computer Engineering
Intelligent Systems and Control Laboratory
404 Engineering South
Stillwater, Oklahoma 74078-5032 USA
Re: Supporting Dr. Jeffrey Tsang for the IEEE CIS Dissertation Award.

Dear Professor Yen:

I am pleased to contribute a supporting letter for the nomination of Dr. Jeffrey Tsang for the IEEE CIS Outstanding PhD Dissertation Award. I served as his external examiner, after meeting him at a conference in 2013. I have been on the faculty of Computer Science at the University at Buffalo for over 25 years; my specialty is Computational Complexity but as partner on Richard Lipton's *Gödel's Lost Letter* blog my knowledge of many things is well maintained.

This dissertation is a great exemplar of "Concrete Mathematics" in the sense of blending continuous and discrete methods. Iterated two-player non-zero-sum games are a conducive subject for this, to be sure, but in this thesis the discrete methods come "ride on top" in a novel way. Namely, considerations from automata theory are employed to segregate the spaces of infinite strategies. This sets up the author's first main technical contribution, which is to smoothen his co-advisor's descriptive concept of "fingerprinting" so that it behaves continuously at boundaries. This enables good definitions of metrics and formulas that can be characterized as volume estimates. The theme is culminated by his showing that flaws in a purely geometrical approach to multidimensional scaling can be solved by blending in a discrete computational process.

The result is a deep investigation into the structure of solution spaces associated to the games, particularly for the salient span of games whose payoff matrix obeys the inequalities of the "Prisoner's Dilemma" pattern. This is conveyed both by substantial numerical analysis and by diagrams of stunning beauty. Their power is shown by a full treatment of the problem of equivalence of strategies. They also establish a strong connection to evolutionary computation. The high quality is maintained over more than 300 pages, plus software implementations (covered in a detailed appendix), and the thesis would make an attractive book publication.

The thesis fits the classic computational-intelligence mold insofar as it evaluates the performance of the game strategies, but its *evocation* is deeper—the diagrams reveal a mathematical landscape on which the strategies' intelligence is founded. The thesis shows how

information structures are channeled by the underlying forms. This impression is bracketed with acknowledging that the thesis is more theory-building than problem-solving, and that the canonical technical problem solved was formulated "in-house" in earlier work by the adviser and colleagues.

On a closer-to-ground level, I can give a short-term appraisal of the dissertation's significance. Iterated games have been the subject of FOCS/STOC papers by Lance Fortnow and Xi Chen and others, while evolutionary game theory makes regular appearances in the journals *Science* and *Nature* (here I am recalling discussion a year ago in a blog post by Umesh Vazirani). Three single-author papers from Tsang's thesis have been accepted this year to conferences dedicated to evolutionary computing. Game strategies under repetition are a major focus of online auctions and areas of resource contention such as file-sharing. Hundreds of applications of the "Prisoner's Dilemma" pattern appear each year; Tsang cites three papers with PD in their title by Harvard biologist Martin Nowak alone. The thesis also has applications to numerical methods in general, including a demonstration that N-times-differentiable functions in some sense densely cover the space of all functions (however non-differentiable) that have $(N + 1)$st-order approximations. I can, however, say definitively that the thesis is an intellectual Renaissance master's achievement blending pure and applied mathematics and computer science, and likely to be a long-term influence on where people seeking new ideas in combining these subjects should first look.

Yours sincerely

Dr. Kenneth W. Regan
(716) 645-4738, `regan@buffalo.edu`