The Length of the Beacon Attraction Trajectory

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Abstract

We study the attraction trajectory of a point under the beacon model. We show that when a point object p is attracted to a point beacon b, inside a simple polygon P, its trajectory is at most $\sqrt{2}$ times the geodesic distance between p and b in P.

1 Introduction

In 2011, Biro et al. [2] introduced the beacon model as a variation of visibility. Let P be a simple polygon containing points p and b, where b represents a *point beacon* that can emit an attractive force on the point object p. The attractive force of beacon b pulls object p directly towards it. This attraction may land p onto an edge e of P. Then p slides on e towards the point h, the orthogonal projection of b on the supporting line of e. Note that h has the shortest Euclidean distance to b among all points on the supporting line of e. Therefore, the movement of p alternates between moving towards b and sliding on edges of P. A point in P is *attracted* by b if its Euclidean distance to b is eventually decreased to 0. The *attraction region* of a beacon b is the set of all points in P that b can attract and can be computed in linear time [1]. Whenever p is attracted to b we can define its *attraction trajectory* denoted by $AT(p,b)$ as the path p takes until it reaches b. Let $SP(p,b)$ denote the shortest path between p and b inside the polygon P. We use $|SP(p,b)|$ and $|AT(p,b)|$ to denote the lengths of these paths. Tan and Kermarrec [4] showed that $|AT(p,b)| \leq 3\,|SP(p,b)|$. In this abstract we improve this bound to $|AT(p,b)| < \sqrt{2}\,|SP(p,b)|$. For additional results on the beacon model see [1, 3].

2 Results

Observation 1: $AT(p,b)$ does not necessarily pass through all the reflex vertices located on the shortest path from p to b in P (see Fig. 1).

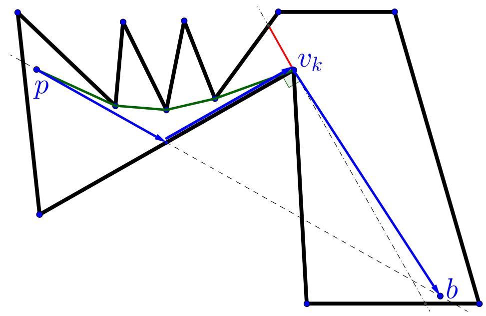

Figure 1: The attraction trajectory does not necessary pass through all reflex vertices in $SP(b,p)$.

Lemma 1. *Let $SP(p,b)$ be the polygonal chain $p, v_1, v_2, ..., v_k, b$ and assume b attracts p. If the edge $\overline{v_i v_{i+1}} \in SP(p,b)$ partitions P into two subpolygons such that b and p are in different subpolygons, then at least one of v_i or v_{i+1} is on $AT(p,b)$. In addition v_k is always on $AT(p,b)$.*

Proof. We omit the proof due to lack of space. □

Observation 2: The attraction trajectory can rotate with an arbitrary degree around b and the length of $AT(p,b)$ can be arbitrarily bigger than the Euclidean distance between b and p (See Fig. 2).

In order to determine the quality of the attraction trajectory we compare its length to the length of $SP(b,p)$ in P.

Theorem 1. $|AT(p,b)| < \sqrt{2}\,|SP(p,b)|$.

Proof. We partition $AT(p,b)$ into maximal subpaths that alternately coincide with and diverge from $S(p,b)$. We then show that each maximal divergent subpath of $AT(p,b)$ is at most $\sqrt{2}$ longer than the corresponding part of $SP(p,b)$. Let v be the first reflex vertex of P such that both $SP(p,b)$ and $AT(p,b)$ pass through v. Let $AT(p,v)$ denotes the part of $AT(p,b)$ between p and v. It is sufficient to show that $|AT(p,v)|\,/\,|\overline{pv}| < \sqrt{2}$ (note that $|\overline{pv}| \leq |SP(p,v)|$). By lemma 1, v is a reflex

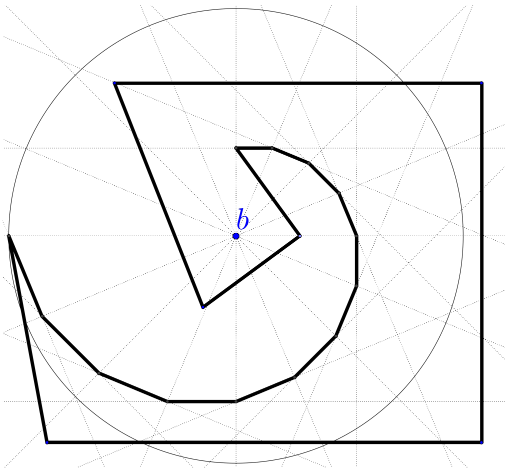

Figure 2: The attraction trajectory my rotates several time around b.

vertex on the shortest path between p and some reflex vertex $v_{i+1} \in SP(p,b)$, where $\overline{v_i v_{i+1}}$ is the first edge that partitions p and b. Therefore, according to shortest path properties, the part of the shortest path between p and v is a convex chain in P. Without loss of generality, assume that p, b and v have relative positions as in Fig. 3 with the following assumptions:
1) The attraction path does not cross $\overline{pv}$ or $SP(p,v)$. 2) Above v is the inside part of P. Now let u be the first intersection of the ray $\overrightarrow{pb}$ with the boundary of P. In the attraction of b, p moves directly towards b until it reaches u then it starts to slide. Consider a polar coordinate system with the reference point b, with the components r (the distance of the point to b) and ϕ (the counter clockwise angle between the axis and a line joining b to the point). During the motion of p from its original position to v, by the definition of the beacon model, the r-coordinate of p monotonically decreases. In addition, we can prove that during each slide in $AT(p,v)$ the ϕ-coordinate does not increase.
Consider an attraction trajectory from p to v (shown in red in Fig. 3). We convert this attraction trajectory to a longer one by considering each slide to end on a right angle with the line going through b and the end of slide (shown in green). Next we convert to an even longer attraction trajectory by forcing each slide to be parallel to the green ones but with an end point located on the edge $\overline{pv}$ (shown in blue in Fig. 4). Note that parts of the blue and the green chains that do not collide form a trapezoid where the angle between the two green segments is greater than $\pi/2$. This guarantees that the blue chain is longer than the green chain. Now consider a triangle with two blue line segments and the portion of $\overline{pv}$ between these two line segments. As the angle between the blue line segments is at least $\pi/2$, the total length of the blue path is at most $\sqrt{2}$ of the length of $\overline{pv}$. $\square$

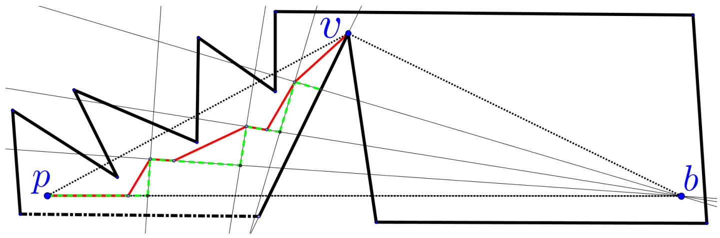

Figure 3: Converting to a longer attraction trajectory.

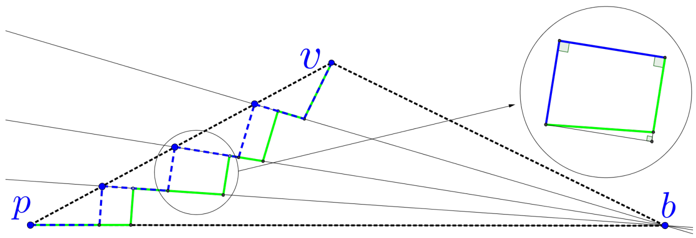

Figure 4: Converting to even a longer attraction trajectory.

References

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