Geometric Hitting Set for Segments of Few Orientations*

Sándor P. Fekete[1], Kan Huang[2], Joseph S. B. Mitchell[2], Ojas Parekh[3], and Cynthia A. Phillips[3]

[1] s.fekete@tu-bs.de, TU Braunschweig, Braunschweig, Germany
[2] {khuang, jsbm}@ams.stonybrook.edu, Stony Brook University, Stony Brook, NY, USA
[3] {odparek, caphill}@sandia.gov, Sandia National Labs, Albuquerque, NM, USA

1 Introduction

Numerous special instances of set cover/hitting set have been studied. Our focus in this paper is on geometric instances that arise in covering (hitting) sets of (possibly overlapping) line segments using the fewest points ("hit points"). We provide several new results on hardness and approximation algorithms.

We are motivated by the path monitoring problem: given a set of trajectories, each a path of line segments in the plane, place the fewest sensors (points) to observe (hit) all trajectories. If the trajectories are on a Manhattan road network, the paths are (possibly overlapping) horizontal/vertical segments.

Our results. We give complexity and approximation results for several problems all in the family of geometric hitting set problems on inputs S of line "segments" of special classes, mostly of fixed orientations. The segments are allowed to overlap arbitrarily. We consider various cases of "segments" that may be bounded (line segments), semi-infinite (rays), or unbounded in both directions (lines). Our main results are:

(1) Hitting lines of 3 slopes in the plane is NP-hard. (Hitting lines of 2 slopes is easy to solve optimally.) Naively, one obtains a 3-approximation for hitting lines of 3 slopes (since one can trivially hit (parallel) lines of one slope optimally). The standard analysis of the greedy algorithm gives an approximation factor of $H(3) = 1 + (1/2) + (1/3) = (11/6)$. We prove that the greedy algorithm in this special case gives an approximation factor of $7/5$.

(2) Hitting vertical lines and horizontal rays is polytime solvable.

(3) Hitting vertical lines and horizontal (even unit-length) segments is NP-hard. A consequence of our proof is that hitting horizontal and vertical unit-length segments is also NP-hard. Further, we prove APX-hardness for hitting horizontal and vertical segments.

(4) Hitting vertical lines and horizontal segments has a $(5/3)$-approximation algorithm. (This problem has a straightforward 2-approximation.)

(5) Hitting pairs of horizontal/vertical segments has a 4-approximation. Hitting pairs having one vertical line and one horizontal segment has a $(10/3)$-approximation. These results are based on LP-rounding. More generally, hitting sets of k segments from r orientations has a $(k \cdot r)$-approximation algorithm.

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Related Work. Hassin and Megiddo [5] considered hitting geometric objects with the fewest lines having a small number of distinct slopes. They gave approximations for the problem of hitting horizontal/vertical segments with the fewest axis-parallel lines (and, more generally, with lines of a few slopes). A recent paper [6] gives a 3-approximation for hitting sets of segments that are "triangle-free". Brimkov et al. [3, 2, 1] have studied the hitting set problem on line segments, including various special cases; they refer to the problem as "Guarding a Set of Segments", or GSS. An important distinction between the GSS and the problems we study here is that the set S of segments we allow includes *overlapping* (or partially overlapping) segments (rays, and lines), while, in the GSS, one assumes that each line segment is maximal in the input set of line segments (the union of two distinct input segments is not a segment).

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