Segmented ODETLAP Compression

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ABSTRACT

We have designed an algorithm of segmented ODETLAP compression for spatial data simplification. It can be more than 3 times as fast as unsegmented ODETLAP compression and use less than 10% more points for similar or better compression errors. When hardware is available, it can also process segments in parallel.

1. INTRODUCTION

Overdetermined Laplacian partial differential equations (ODETLAP) is a spatial data approximation and compression method [1]. The motivation of ODETLAP approximation is the smooth approximation of digital elevation models from contours, in which a method like natural neighbor interpolation produces terracing artifact. There are two objectives: minimizing the smoothness measure of the approximation, which is measured by the 2D integration of the Laplacian, and minimizing the approximation error of known points. The input of ODETLAP approximation is a set of known points $S = \{(x_i, y_i)\}_k$ and values $f(x_i, y_i), i = 1 \dots k$, and the output is the approximated values of all grid points $u(x_i, y_i), i = 1 \dots n$. The algorithm is to build and solve an overdetermined system of two groups of equations. The first group is Laplace's equations for all grid points

$$R(u(x-1, y)+u(x+1, y)+u(x, y-1)+u(x, y+1)-4u(x, y) = 0),$$

where R is a weight called the smoothing factor. The second group is known point equations $u(x, y) = f(x, y)$. The system has $n + k$ equations and n unknowns, and is solved by linear least squares.

ODETLAP compression is based on ODETLAP approximation [5]. The input is a point dataset and the output is a set of known points and values S. The basic objective of ODETLAP compression is to find an S of limited size, such that the maximum absolute point error of its ODETLAP approximation to the dataset is minimized. The basic algorithm starts with a small subset of the dataset as an initial S, and iteratively selects new points from the dataset. In each iteration, it computes the ODETLAP approximation of S, and adds to S the data point with the greatest absolute approximation error. The algorithm stops when the size or approximation error of S reaches a threshold. Besides compression, the method is also useful for the progressive transmission of large datasets. ODETLAP approximation is reasonably fast, but ODETLAP compression is slow with thousands or tens of thousands iterations. In this paper, we try to accelerate ODETLAP compression by exploring the local behavior of ODETLAP approximation.

To accelerate ODETLAP approximation, Stookey [4] parallelized it on an IBM Blue Gene/L by dividing a grid into overlapping patches. For example, to approximate a terrain, the method divides the grid and known points into overlapping patches of size 100×100, whose lower left corners are at $(50i, 50j)$, $i, j = 0, 1, \dots$ Then it computes an approximation for each patch and merges the results using bilinear interpolation. The patches are grouped into blocks that are processed in parallel. To save time and memory for ODETLAP approximation, Li [2] used a method that divides a grid into two overlapping sets of boxes. Then it computes an approximation for each box and merges the results from the two sets using weighted average.

Table 1: Unsegmented results

Points	Time	AVGEE	RMSEE	MAXEE
6386	52m26s	12.0	15.6	50.0

Mitášová and Mitáš [3] developed a segmentation procedure for the interpolation of large datasets using completely regularized splines. The method is based on the local behavior of the interpolation function. It divides a grid into square segments so that the number of known points in each segment and its neighborhood is less than a threshold. Then it computes an interpolation for each segment from the points in its neighborhood of 3×3, 5×5 or more segments, so that the number of points is more than a threshold.

2. SEGMENTED ODETLAP COMPRESSION

Solving large linear systems is time-consuming but can be accelerated by parallel processing on GPU. We implemented ODETLAP approximation using the Cusp library, which is based on the Thrust library. On our server, the speedup of ODETLAP approximation is about 8 times using an NVIDIA Tesla K20Xm, over using a single thread of an Intel Xeon E5-2687W. With GPU-accelerated ODETLAP approximation, we can afford to process bigger datasets, and to add one point in each iteration of the greedy point selection method. However, it is still a time-consuming process. For example, Figure 1 shows a 600×600 DEM down-sampled from NED 1×1 degree block n43w074. Point values are in integer meters and have a range of $[-1, 1138]$.

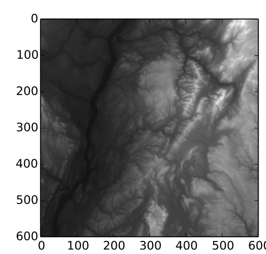
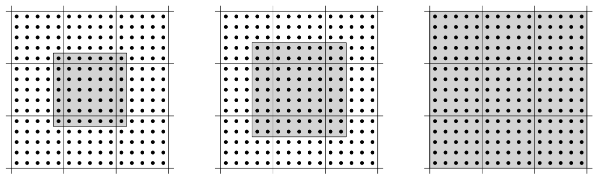


Figure 1: Sample dataset.

Table 1 shows the number of selected points, running time and compression errors of adding one point per iteration to an initial set of 40×40 regular points at positions $(15i + 7, 15j + 7)$, until the maximum absolute elevation error (MAXEE) is less than 50 meters. The other errors are average absolute elevation error (AVGEE) and root-mean-square elevation error (RMSEE) in meters. The smoothing factor of ODETLAP approximation is $R = 0.01$.

ODETLAP approximation shows local behavior in that the influence of a known point decreases with the increase of distance. Instead of segmented approximation, we designed segmented compression such that adding a point only requires an approximation for a single segment.



(a) (b) (c)

Figure 2: Neighborhood types.

2.1 Algorithm

The main idea of the algorithm is to divide an $nrows \times ncols$ dataset into $\frac{nrows}{SS} \times \frac{ncols}{SS}$ segments of size $SS \times SS$, and then compress each segment in association with neighboring segments. Given a segmented dataset, it selects points from each segment until the maximum absolute approximation error of each segment is less than a threshold. The approximation of a segment is computed as part of the approximation of its neighborhood. A segment still needs processing if either its maximum approximation error is not less than the threshold, or new points are selected in its neighborhood from other segments. When a point is selected, all segments whose neighborhood contains it will need processing. To maintain uniform progress among all segments, they are processed in a round-robin manner. The details are shown in Algorithm 1.

Algorithm 1: Segmented compression

Data: a segmented dataset
Result: a set S of selected points
add the center of each segment to S;
mark all segments as needing processing;
while *there are segments that need processing* **do**
    for *each segment s that needs processing in an order* **do**
        while *s needs processing for up to a number of iterations* **do**
            compute the ODETLAP approximation of s's neighborhood;
            if *the maximum absolute error in s is less than a threshold* **then**
                mark s as not needing processing;
            else
                add the worst point p in s to S;
            for *each other segment t in s's neighborhood* **do**
                if *p is in t's neighborhood* **then**
                    mark t as needing processing;

The parameters are SS: segment size; NT: neighborhood type; $MAXITER$: the maximum number of iterations in processing a segment; and $ORDER$: the order of processing the segments that need processing.

2.2 Experiments

We considered three neighborhood types: (a) one extra point wide, (b) $\frac{SS}{2}$ extra points wide, and (c) one extra segment wide. Figure 2 shows each type of neighborhood as a gray box when $SS = 5$.

In experiments, we set $SS = 15$ and the maximum approximation error threshold of each segment to 50. The initial point set consists of the center of each segment, or 40×40 regular points. The smoothing factor of ODETLAP approximation $R = 0.01$. Table 2 shows the results of the algorithm using different NT's, with $MAXITER = 1$ and $ORDER$ being row-column order. The approximation of a dataset consists of the approximation of each segment. The table also shows the inflation of the number of selected points and the speedup of running time. As neighborhood size increases from (a) to (c), the inflation, speedup and errors all decrease.

Table 2: Segmented results (NT)

NT	Points	Inflation	Time	Speedup	AVGEE	RMSEE	MAXEE
(a)	8071	1.26×	6m45s	7.76×	12.0	15.6	50.0
(b)	7483	1.17×	9m26s	5.56×	11.8	15.3	50.0
(c)	7014	1.10×	20m14s	2.59×	11.8	15.3	50.0

Table 3: Segmented results ($MAXITER$)

M.I.	Points	Inflation	Time	Speedup	AVGEE	RMSEE	MAXEE
3	6915	1.08×	16m40s	3.15×	11.8	15.3	49.9
4	6898	1.08×	15m53s	3.30×	11.8	15.3	50.0
5	6888	1.08×	15m40s	3.35×	11.9	15.4	50.0

Table 3 shows the results using different $MAXITER$'s, with NT being (c) and $ORDER$ being row-column order. In general, as $MAXITER$ increases, the speedup decreases but converges quickly, while the other results are similar.

Table 4 shows the results with NT being (c), $MAXITER = 4$ and $ORDER$ being random order. Random order is slightly faster than row-column order.

Table 4: Segmented results ($ORDER$)

Points	Inflation	Time	Speedup	AVGEE	RMSEE	MAXEE
6821	1.07×	15m28s	3.39×	11.9	15.4	50.0

3. CONCLUSIONS

We have designed segmented ODETLAP compression. For the sample dataset and a maximum error of 50, it is more than 3 times as fast as unsegmented compression. Average and RMS errors are sightly better, but the number of selected points is about 7% larger. Because a GPU is less efficient with a smaller problem size, the relative speedup is greater if all computation is on CPU. The algorithm also works better for more unbalanced datasets. Besides, when hardware is available, segments can be processed in parallel.

Acknowledgement. This research was partially supported by NSF grant IIS-1117277 and CAPES (Ciência sem Fronteiras).

4. REFERENCES

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