A Geometric Matching Model Based on Earth Mover's Distance and Its Applications in Computer Vision

Jianxu Chen, Mark S. Alber, and Danny Z. Chen

Department of Computer Science and Engineering and Department of Applied and Computational Mathematics and Statistics, University of Notre Dame, USA

1 Background

Computing an optimal matching of two sets of geometric objects, such as points, polygons, or general shapes, is a fundamental problem in computational geometry and has broad applications in computer vision and computer graphics. The matching of point sets (e.g., for registration of point sets) has been studied for decades, and many algorithms have been proposed for both rigid and non-rigid matching [7]. For polygons or polylines, the matching problem could be tackled by matching features, such as the angle turning function [11]. When the objects are general shapes, e.g., collections of simply connected regions, the matching problem is still being actively studied.

2 Methodology

We study a geometric matching model based on the Earth Mover's Distance (EMD) [9] for solving the matching problem of general shapes, under the following assumptions: (1) the objects may divide or merge, (2) some objects may have no correspondence, and (3) the optimality can be application-dependent. Specifically, given two sets of shapes, $P = \{P_1, P_2, \ldots, P_n\}$ and $Q = \{Q_1, Q_2, \ldots, Q_m\}$, where P_i (resp., Q_j) is of size u_i (resp., v_j), we want to find the optimal correspondence between P and Q, with respect to an optimality measure.

The EMD model can be interpreted as the minimal effort for moving piles of dirts of various amounts, as much as possible, into a set of holes of various volumes. A key component of the EMD model is the *ground distance*, i.e., the cost D_{ij} for distributing one unit of dirt from a pile P_i to a hole Q_j. This is a multiple-to-multiple matching model. The essential problem of the EMD matching model is a transportation problem, and thus can be solved in polynomial time. We present two real applications of the EMD model, in which the computation of the ground distance can be integrated with techniques in computational geometry, statistics, or machine learning. Our proposed method achieves considerable improvement over the state-of-the-art algorithms for these applications.

3 Applications

The EMD matching problem has many applications in computer vision, such as image registration, image retrieval, shape morphing, and object tracking. We demonstrate the effectiveness of the EMD matching model in two imaging problems: bacteria tracking and iris recognition.

Bacteria tracking is a crucial step for various biological and biomedical studies [6]. Given a sequence of images captured in time-lapse experiments, the objective is to detect and track the motion of each bacterial cell in the image sequence. In the literature, the bacteria tracking problem can be approached, for example, by bipartite graph matching [5] or min-cost flow optimization [8], to match cells in every two consecutive image frames or find the optimal correspondence in the whole sequence in a graphical model [10]. However, most of the known algorithms either cannot handle object division/fusion, or need to solve an NP-hard integer programming problem.

We apply the EMD matching model to the tracking of two types of bacteria, *Myxococcus Xanthus* and *Pseudomonas Aeruginosa.* They are both bacteria with rod-like shapes, but differ in two aspects. First, *M. Xanthus* may divide, but *P. Aeruginosa* cannot. Second, they have different motion patterns. *M. Xanthus* mostly glide forward or backward along the body orientation, but rarely move in the direction normal to the body orientation. But, *P. Aeruginosa* can move freely, and may be restricted only due to the tightness of the neighboring cells. We adopt two different definitions of *ground distance* based on curve matching and acyclic order-preserving assignment for *M. xanthus* [1, 2] and *P. Aeruginosa*, respectively.

The iris recognition problem arises from the lack of human interpretability in current biometric techniques [4], which could be a barrier to the official employment of iris recognition in forensics. We apply the EMD matching model in an iris recognition framework based on visible features. Basically, given two iris images and the detected visible features (called crypts) on each image, we match the detected features and measure the similarity in order to determine whether the two images are from the same eye [3]. In this scenario, the ground distance is defined as a combination of systemic difference and Hausdorff distance.

References

[1] J. Chen, C. Harvey, M. S. Alber, and D. Z. Chen. A matching model based on earth mover's distance for tracking myxococcus xanthus. In *17th International Conference on Medical Image Computing and Computer-Assisted Intervention (MICCAI)*, pages 113–120, 2014.

[2] J. Chen, S. Mahserejian, M. S. Alber, and D. Z. Chen. A hybrid approach for segmentation and tracking of myxococcus xanthus swarms. In *18th International Conference on Medical Image Computing and Computer-Assisted Intervention (MICCAI)*, pages 284–291, 2015.

[3] J. Chen, F. Shen, D. Z. Chen, and P. J. Flynn. Iris recognition based on human-interpretable features. In *IEEE International Conf. on Identity, Security and Behavior Analysis (ISBA)*, pages 1–6, 2015.

[4] J. Daugman. How iris recognition works. *IEEE Trans. on Circuits and Systems for Video Technology*, 14(1):21–30, 2004.

[5] X. Liu, C. W. Harvey, H. Wang, M. S. Alber, and D. Z. Chen. Detecting and tracking motion of myxococcus xanthus bacteria in swarms. In *15th International Conference on Medical Image Computing and Computer-Assisted Intervention (MICCAI)*, pages 373–380, 2012.

[6] E. Meijering, O. Dzyubachyk, and I. Smal. Methods for cell and particle tracking. *Methods Enzymol*, 504(9):183–200, 2012.

[7] A. Myronenko and X. Song. Point set registration: Coherent point drift. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, 32(12):2262–2275, 2010.

[8] D. Padfield, J. Rittscher, and B. Roysam. Coupled minimum-cost flow cell tracking for high-throughput quantitative analysis. *Medical Image Analysis*, 15(4):650–668, 2011.

[9] Y. Rubner, C. Tomasi, and L. J. Guibas. The earth mover's distance as a metric for image retrieval. *International Journal of Computer Vision*, 40(2):99–121, 2000.

[10] M. Schiegg, P. Hanslovsky, C. Haubold, U. Koethe, L. Hufnagel, and F. A. Hamprecht. Graphical model for joint segmentation and tracking of multiple dividing cells. *Bioinformatics*, 31(6):948–956, 2014.

[11] M. Tănase, R. C. Veltkamp, and H. Haverkort. Multiple polyline to polygon matching. In *International Symposium on Algorithms and Computation*, pages 60–70, 2005.