Conflict-free Covering*

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Abstract

Let $P = \{C_1, C_2, \ldots, C_n\}$ be a set of color classes, where each color class C_i consists of a set of points. In this paper, we address a family of covering problems in which each color class must be covered but no two points from the same color class are allowed to be covered by the same geometric object. We prove that the problems in this family are NP-complete (or NP-hard) and offer several constant-factor approximation algorithms. The proofs will be omitted due to limited space. Portions of these results can be found in the full paper that was accepted to and presented at CCCG 2015 [1]. Other portions were accepted to and will be presented at ISAAC 2015 [2]. The entire list of references can also be found in the full papers.

1 Introduction

Let $P = \{C_1, C_2, \ldots, C_n\}$ be a set of color classes, where each color class C_i consists of a set of points. We address a family of covering problems in which each color class must be covered but no two points from the same color class are allowed to be covered by the same geometric object (a connected entity). We explore problems in which at least one point of each color class must be covered and problems in which exactly one point of each color class must be covered. We call an interval on the x-axis that contains at most one point from each color class a *conflict-free* interval (or CF-interval for short). Unless otherwise specified, we assume $|C_i| = 2 \; \forall \; i$. We consider the following problems.

Problem 1 (Covering color classes with CF-intervals) *Let P be on a line. Find a minimum-cardinality set $\mathcal{I}$ of* arbitrary *CF-intervals, such that at least one point from each color class is covered by an interval in $\mathcal{I}$.*

Problem 2 (Covering color classes with a given set of CF-intervals) *Let P be on a line. Given a set $\mathcal{I}$ of CF-intervals, find a minimum-cardinality set $\mathcal{I}' \subseteq \mathcal{I}$ (if it exists), such that at least one point from each color class is covered by an interval in $\mathcal{I}'$.*

Problem 3 (Covering color classes with unit length intervals) *Let P be on a line. Decide whether or not there exists a set of unit length intervals, $\mathcal{I}$, such that exactly one point from each color class is covered by an interval in $\mathcal{I}$.*

Problem 4 (Covering color classes with unit length intervals) *Let P be on a line. Find a minimum-cardinality set $\mathcal{I}$ of unit length intervals (assuming a feasible solution exists), such that exactly one point from each color class is covered by an interval in $\mathcal{I}$.*

Problem 5 (Covering color classes with intervals of arbitrary length) *Let P be on a line. Find a minimum-cardinality set $\mathcal{I}$ of intervals of arbitrary length, such that exactly one point from each color class is covered by an interval in $\mathcal{I}$.*

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Problem 6 (Covering color classes with arbitrary unit squares) *Let P be in the Euclidean plane and let each color class C_i consist of a vertically or horizontally unit separated pair of points. Find a minimum-cardinality set $\mathcal{S}$ of axis-aligned unit squares (assuming a feasible solution exists), such that exactly one point from each color class is covered by a square in $\mathcal{S}$.*

Problem 7 (Covering color classes with a convex polygon) *Let P be in the Euclidean plane and let each color class C_i consist of either a pair or a triple of points. Decide whether or not there exists a convex polygon Q such that Q contains exactly one point from each color class.*

Problem 8 (Covering color classes with a convex polygon) *Let P be in the Euclidean plane and let each color class C_i consist of a pair of points. Maximize the number of color classes covered by a convex polygon Q such that Q contains exactly one point from each covered color class.*

2 Related work

As far as we know, the first to consider a "multiple-choice" problem of this kind were Gabow et al. [7], who studied the following problem. Given a directed acyclic graph with two distinguished vertices s and t and a set of k pairs of vertices, determine whether there exists a path from s to t that uses at most one vertex from each of the given pairs. They showed that the problem is NP-complete. A sample of additional graph problems of this kind can be found in [4, 8, 9]. The first to consider a problem of this kind in a geometric setting were Arkin and Hassin [5], who studied the following problem. Given a set V and a collection of subsets of V, find a cover of minimum diameter, where a cover is a subset of V containing at least one representative from each subset. They also considered the multiple-choice dispersion problem, which asks to maximize the minimum distance between any pair of elements in the cover. They proved that both problems are NP-hard and gave $O(1)$-approximation algorithms. Recently, Arkin et al. [3] considered the following problem. Given a set S of n pairs of points in the plane, color the points in each pair by red and blue, so as to optimize the radii of the minimum enclosing disk of the red points and the minimum enclosing disk of the blue points. In particular, they consider the problems of minimizing the maximum and minimizing the sum of the two radii. In another recent paper, Consuegra and Narasimhan [6] consider several problems of this kind.

3 Our results

We show that all of Problems 1-8 are NP-complete (or NP-hard) [1, 2]. Additionally, we give a 2-approximation for Problem 1, a 4-approximation for Problem 2 and a 6-approximation for Problem 6 [1].

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