EXTENDING THE POWER OF NUMERICAL QUANTIFIERS

by

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Abstract

The power of the numerical quantifiers available in SNePS [Shapiro 79] has recently been increased by allowing their application to formulas with multiple consequents. This note describes such extension and shows examples of their use.

Numerical quantifiers could only be applied to formulas of the form ${}_n\exists_i^j \bar{x}(P_1(\bar{x}),..,P_k(\bar{x}):Q(\bar{x}))$ where $k \geq 0$ and $\bar{x}$ represents a sequence of variables each of which is free in at least one of $P_1(\bar{x})$, .. ,$P_k(\bar{x})$,$Q(\bar{x})$. The above expression means that of the n combinations of individuals satisfying $P_1(\bar{x})$&..&$P_k(\bar{x})$ there are at least i and at most j which also satisfy $Q(\bar{x})$.

The implementation of forward inference introduced the possibility of deriving multiple consequents in rules whose main connectives are v-> and &->. In this way the non-standard connectives available in SNePS (v->, &->, ${}_n X_i^j$, ${}_n \Theta_i$) are fully operational with multiple consequents. Under this line of reasoning it was natural to allow the use of numerical quantifiers in rules with multiple consequents, and that is what this note is all about.

Numerical quantifiers have now the following syntax:

1. ${}_n\exists_i^j\bar{x}(P_1(\bar{x}),..,P_k(\bar{x}):Q_1(\bar{x}),..,Q_m(\bar{x}))$;

2. ${}_n\exists_i\bar{x}(P_1(\bar{x}),..,P_k(\bar{x}):Q_1(\bar{x}),..,Q_m(\bar{x}))$;

3. ${}_n\exists^j\bar{x}(P_1(\bar{x}),..,P_k(\bar{x}):Q_1(\bar{x}),..,Q_m(\bar{x}))$.

In the above expressions $k\geq\emptyset$, $m>\emptyset$ and $\bar{x}$ represents a sequence of variables each of which is free in at least one of $P_1(\bar{x}),..,P_k(\bar{x}),Q_1(\bar{x}),..,Q_m(\bar{x})$. The meaning of formula in line 1 is that of the n combinations of individuals satisfying $P_1(\bar{x})\&..\&P_k(\bar{x})$ at least i and at most j of these combinations also satisfy the conjunction $Q_1(\bar{x})\&..\&Q_m(\bar{x})$; rule in line 2 means that of the n combinations of individuals satisfying $P_1(\bar{x})\&..\&P_k(\bar{x})$ at least i of them also satisfy $Q_1(\bar{x})\&..\&Q_m(\bar{x})$, while rule in line 3 means that at most j of these combinations also satisfy $Q_1(\bar{x})\&..\&Q_m(\bar{x})$.

If a rule of type (1) exists in the network and if the user originates the creation of a CH-process working on an instance of such a rule, either by asking for $Q_i(\bar{a})$, (where $1\leq i\leq m$ and $\bar{a}$ is a substitution instance of $\bar{x}$) or by ADDing to the network a node matching $P_j(\bar{x})$ $(1\leq j\leq k)$, subgoal derivations of $P_1(\bar{x}),..,P_k(\bar{x}),Q_1(\bar{x}),..,Q_m(\bar{x})$ are begun. One of the following cases will occur:

1. j substitution instances of $\bar{x}$ are found for which $P_1(\bar{x})\&..\&P_k(\bar{x})\&Q_1(\bar{x})\&..\&Q_m(\bar{x})$. If this occurs let $Q'(\bar{x})$ be ${}_mX_\emptyset^{m-1}(Q_1(\bar{x}),..,Q_m(\bar{x}))$;
2. n-i substitution instances of $\bar{x}$ are found for which

$P_1(\bar{x})\&..\&P_k(\bar{x})\&(\sim Q_1(\bar{x})v..v\sim Q_m(\bar{x}))$. If this occurs let $Q'(\bar{x})$ be ${}_{m}X^{m}_{m}(Q_1(\bar{x}),..,Q_m(\bar{x}))$;

3. If none of the above cases occurs the rule is incapable of deriving any of the consequents with the existing data.

If either case 1 or 2 occurs the process implementing this rule changes itself into one implementing the rule $(P_1(\bar{x}),..,P_k(\bar{x}))$ &-> $Q'(\bar{x})$, retains the relevant data already accumulated and continues processing. Which means that for every new substitution instance of $\bar{x}$, say $\bar{x}_o$, such that $P_1(\bar{x}_o)\&...\&P_k(\bar{x}_o)$ the rule $Q'(\bar{x}_o)$ will be derived and a CH-process will be created to use it. This means that in case 1 this CH-process (working on the instance $Q'(\bar{x}_o)$ of the rule $Q'(\bar{x})$) will set up subgoal derivations for $Q_1(\bar{x}_o),..,Q_m(\bar{x}_o)$ and if m-1 of them are ever found it will deduce the negation of the remainder; in case 2 the CH-process working on the rule instance $Q'(\bar{x}_o)$ will deduce $Q_1(\bar{x}_o)\&..\&Q_m(\bar{x}_o)$.

The following runs show examples of the two cases discussed above:

====== <u>Run No.1:</u> ======

```
** (SURFACE (BUILD ETOT 5 EMIN 2 EMAX 2 PEVB $X
*                 &ANT (BUILD MEMB *X CLASS FACULTY)
*                 CQ   ((BUILD MEMB *X CLASS AI/ TEACHER)
*                      (BUILD AGT *X R IN OBJ AI/ MEETING))))
OF THE 5 V1, SUCH THAT
   V1 IS A FACULTY MEMBER
      THERE ARE EXACTLY 2 SUCH THAT
          V1 IS IN THE AI MEETING AND
            V1 IS AN AI TEACHER
(DUMPED)
399 MSECS

** (SURFACE (BUILD MEMB PAT CLASS FACULTY))

PAT IS A FACULTY MEMBER
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```
(DUMPED)
514 MSECS

** (SURFACE (ADD MEMB STU CLASS FACULTY))


STU IS A FACULTY MEMBER
(DUMPED)
401 MSECS

** (SURFACE (ADD MIN Ø MAX Ø ARG
*                ARG (BUILD MEMB LARRY CLASS AI/ TEACHER)))


LARRY IS NOT AN AI TEACHER
(DUMPED)
319 MSECS

** (SURFACE (ADD MIN Ø MAX Ø
*                ARG (BUILD MEMB TONI CLASS AI/ TEACHER)))


TONI IS NOT AN AI TEACHER
(DUMPED)
319 MSECS

** (SURFACE (ADD MEMB LARRY CLASS FACULTY))


LARRY IS A FACULTY MEMBER
(DUMPED)
145 MSECS

** (SURFACE (ADD MEMB TONI CLASS FACULTY))


TONI IS A FACULTY MEMBER
(DUMPED)
144 MSECS

** (SURFACE (ADD MIN Ø MAX Ø
*                ARG (BUILD AGT PAT R IN OBJ AI/ MEETING)))

SINCE
STU IS A FACULTY MEMBER

WE INFER
STU IS IN THE AI MEETING AND
STU IS AN AI TEACHER


PAT IS NOT IN THE AI MEETING AND
STU IS AN AI TEACHER AND
STU IS IN THE AI MEETING
(DUMPED)
16Ø6 MSECS
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```
** (SURFACE (ADD MEMB NICK CLASS FACULTY))

SINCE
NICK IS A FACULTY MEMBER

WE INFER
NICK IS IN THE AI MEETING AND
NICK IS AN AI TEACHER


NICK IS A FACULTY MEMBER AND
NICK IS AN AI TEACHER AND
NICK IS IN THE AI MEETING
(DUMPED)
1393 MSECS
```

====== Run No.2: ======

```
** (SURFACE (BUILD ETOT 5 EMIN 1 EMAX 2 PEVB $X
*                     &ANT (BUILD MEMB *X CLASS FACULTY)
*                     CQ   ((BUILD MEMB *X CLASS AI/ TEACHER)
*                          (BUILD AGT *X R IN OBJ AI/ MEETING)))
OF THE 5 V1, SUCH THAT
   V1 IS A FACULTY MEMBER
      THERE ARE AT LEAST 1 AND AT MOST 2 SUCH THAT
             V1 IS IN THE AI MEETING AND
               V1 IS AN AI TEACHER
(DUMPED)
426 MSECS

** (SURFACE (BUILD AGT STU R IN OBJ AI/ MEETING))

STU IS IN THE AI MEETING
(DUMPED)
520 MSECS

** (SURFACE (BUILD MEMB TONI CLASS FACULTY))

TONI IS A FACULTY MEMBER
(DUMPED)
66 MSECS

** (SURFACE (BUILD MEMB NICK CLASS FACULTY))

NICK IS A FACULTY MEMBER
(DUMPED)
67 MSECS

** (SURFACE (BUILD MEMB STU CLASS AI/ TEACHER))

STU IS AN AI TEACHER
(DUMPED)
70 MSECS

** (SURFACE (BUILD MEMB NICK CLASS AI/ TEACHER))
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```
NICK IS AN AI TEACHER
(DUMPED)
73 MSECS

** (SURFACE (ADD AGT NICK R IN OBJ AI/ MEETING))


NICK IS IN THE AI MEETING
(DUMPED)
146 MSECS

** (SURFACE (ADD MEMB STU CLASS FACULTY))

SINCE
TONI IS A FACULTY MEMBER

WE INFER
AT MOST 1 OF THE FOLLOWING
   1 ) TONI IS IN THE AI MEETING
   2 ) TONI IS AN AI TEACHER


STU IS A FACULTY MEMBER
(DUMPED)
1566 MSECS

** (SURFACE (ADD MEMB LARRY CLASS FACULTY))

SINCE
LARRY IS A FACULTY MEMBER

WE INFER
AT MOST 1 OF THE FOLLOWING
   1 ) LARRY IS IN THE AI MEETING
   2 ) LARRY IS AN AI TEACHER


LARRY IS A FACULTY MEMBER
(DUMPED)
681 MSECS

** (SURFACE (ADD MEMB PAT CLASS FACULTY))

SINCE
PAT IS A FACULTY MEMBER

WE INFER
AT MOST 1 OF THE FOLLOWING
   1 ) PAT IS IN THE AI MEETING
   2 ) PAT IS AN AI TEACHER


PAT IS A FACULTY MEMBER
(DUMPED)
1150 MSECS

** (SURFACE (ADD AGT TONI R IN OBJ AI/ MEETING))
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```
SINCE
TONI IS IN THE AI MEETING

WE INFER
TONI IS NOT AN AI TEACHER


TONI IS IN THE AI MEETING AND
TONI IS NOT AN AI TEACHER
(DUMPED)
836 MSECS

** (SURFACE (ADD MIN Ø MAX Ø
*                 ARG (BUILD MEMB PAT CLASS AI/ TEACHER)))


PAT IS NOT AN AI TEACHER
(DUMPED)
8Ø9 MSECS

** (SURFACE (ADD AGT LARRY R IN OBJ AI/ MEETING))

SINCE
LARRY IS IN THE AI MEETING

WE INFER
LARRY IS NOT AN AI TEACHER


LARRY IS IN THE AI MEETING AND
LARRY IS NOT AN AI TEACHER
(DUMPED)
846 MSECS
```

To use numerical quantifiers with multiple consequents input the file (UPDATE CSDJOAO) after loading (or inputting) from the SNePS files all the functions necessary for the run.

Comments, suggestions and complaints are most welcome.

References

1. Shapiro S., "Numerical Quantifiers and their use in Reasoning with Negative Information", _Proc. IJCAI-79_, pp.791-796.