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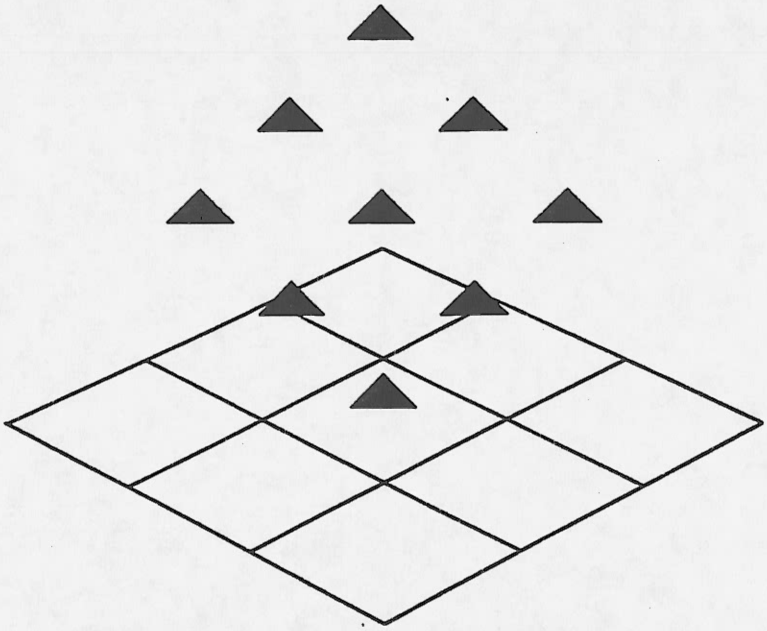

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Representations of Collections in a Propositional Semantic Network

Sung-Hye Cho*
Department of Computer Science
State University of New York at Buffalo
Buffalo, NY 14260
cho@cs.buffalo.edu

Abstract

This paper describes representations of plural entities, collections, in SNePS, the Semantic Network Processing System. The ability to represent collections is crucial for knowledge representation formalisms, as they occur in natural language utterances as often as singular entities. The objective of this research is to provide intensional knowledge representation schemata for collections of various types in SNePS, in order to facilitate inferences about collections and their constituents. The significant differences between sets and collections are described from the viewpoint of intensional representation. Important representational issues concerning propositions involving collections are also discussed. We show representations of several propositions with collections, which demonstrate the ability to represent different interpretations of collection predicates, coreferential collections, collection sizes, and variable collections as used in planning. Some aspects of reasoning about collections are also suggested as areas of future work.

1 Introduction

Human reasoning often involves plural entities as well as singular entities. Plural expressions are encountered frequently in English sentences, such as *All who left* wore their sunglasses and *John and Mary* carried *the dishes in the box* to the kitchen. In this paper, we will refer to a definite plural entity as a *collection* and the individual members of a collection as its *constituents*. We are concerned here with the important problem of representing collections in a propositional representation formalism such that inferences about collections and their constituents can be made.

We note that a collection-constituent hierarchy is different from a generalization hierarchy, which has been used in many knowledge representation systems. A generalization hierarchy is based on attributes, as in Dogs are animals, where the subclass has more attributes than its superclass. On the other hand, a collection hierarchy focuses on the size of collections involved, as in Some of those who gathered brought potluck dishes (Figure 5 d), where the subcollection is indeed smaller than the supercollection.

We review existing set-based[†] representational schemata for collections and discuss why sets are inappropriate as a basis of fully *intensional* representation of collections. We present representations of collections in SNePS, the Semantic Network Processing System [6, 7]. Our objective in developing these representations is to provide a solid framework for modeling a cognitive agent with the ability to: (i) represent both collective and distributive interpretations of a proposition involving collections, (ii) reason about a collection as a whole as well as its individual constituents, (iii) handle coreferential collections, (iv) represent collections as variables, and (v) represent the size of a collection, if known. We demonstrate the effectiveness of the representational schemata by representing several instances of propositions involving collections.

This paper is organized as follows. Section 2 discusses significant differences between sets and collections from the standpoint of intensional representation. Other problems in representation such as different reading methods and variable collections are described in Section 3. Section 4 presents representational schemata for collections in SNePS and shows their expressibility through numerous examples. In Section 5, I suggest some problems to be handled in reasoning about collections as areas of future work.

2 Sets as Representations of Collections

We focus in this section on characteristics of sets as a basis of intensional representation of collections and discuss why they are inappropriate for this purpose.

Constant Collections as Intensional Entities When a constant collection is described by listing all of its constituents, any change in its constituents will result in a different collection. However, consider a constant collection introduced by a denoting phrase which does not specify the exact individual constituents of the collection. For such a collection, I claim that we do not think of a new collection when there is a change in its constituents.

In fact, a denoting phrase merely provides a description for a concrete collection from which we conceive an entity expressed by the words of the denoting phrase, not by the constituents of the collection. In addition, any further knowledge about such a collection, including even the knowledge regarding its specific constituents, makes the description more detailed. Therefore, any changes in the constituents of such a collection should be interpreted as changes in its description, not as a creation of a new collection.

For example, consider the following context consisting of six sentences:

There were *cookies on the table*$_1$.

John ate one of them.

The cookies on the table$_2$ were chocolate chip cookies.

Mary had *some of the cookies*$_3$.

When John went back, there were none left.

The cookies$_4$ were homemade.

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[*] graduate student

[†] In this paper, *set* is mathematical terminology which caused many paradoxes that were settled after ZF Axiom System.

Assume that we are using sets to represent collections (numbered 1, 2, 3, and 4) in the above context. Clearly, *the cookies on the table*$_2$ has one less constituent than *cookies on the table*$_1$ has. However, in one's intension, these two collections correspond to a single collective entity whose description has been slightly modified. Using set representations would force us to creat a new set for *the cookies on the table*$_2$, resulting in multiple representations of a single intensional entity. In addition, a related but different difficulty arises after the fifth sentence, which indicates that the collection is now empty. To represent *the cookies*$_4$, we have to introduce an intensional entity with the empty set as its corresponding extension. We may adopt the notion of intensional sets[4]. In so doing, however, we completely lose the concreteness inherited from the collection *cookies on the table*$_1$.

The cardinality of a set is a unique, well-defined number, be it finite or not. It is impossible to have only one representation for *cookies on the table*$_1$ in the above context because of this property of sets. In addition, there are many collections whose sizes are extremely ill-defined, like *some of the cookies*$_3$. Therefore, the size of a collection need to be treated as a description of the collection, not as part of representation.

Coreferential Collections In set theory, two sets are identical if and only if they have the same elements. On the contrary, when two constant collections are introduced by different denoting phrases but actually consist of the identical constituents, the two collections are not always treated as equivalent entities. For example, consider the following context:

> *Those two lab assistants during morning hours* were energetic and friendly.
>
> But *those two lab assistants during evening hours* were too tired to help anyone.
>
> During the whole day, *John and Mary* were the only assistants in the lab full of students asking for help.

It is obvious that the three collections, *those two lab assistants during morning hours*, *those two lab assistants during evening hours* and *John and Mary* have identical constituents. Yet, we do not want to treat *those two lab assistants during morning hours* and *those two lab assistants during evening hours* as identical entities because of their different properties. Notice that the above context could be developed into a problem which is similar to the classical problem of handling the cardinality in fully intensional knowledge representation [1, 4].

Conclusions The difficulties of using sets as collection representations are mainly due to the diversity of collections as intensional entities, as opposed to singular entities. This motivates the development of representational schemata for collections which is not set-based so that collections can be represented as intentional entities, not as simple aggregations of constituents.

3 Representational Issues

This section describes some issues that need to be considered for designing representational schemata for collections.

Distributive vs Collective Interpretations There are two different ways of reading some sentences containing descriptions of collections: the distributive reading and the collective reading [2, 3, 9]. The semantic interpretation of both reading methods involves one collection; on the distributive reading, the action or predicate is spread over the singular individuals making up the collection; on the collective reading, it is a straightforward predicate of the collection. For example, Pick up the green blocks could be either Pick up the green blocks one by one (distributive) or Pick up the green blocks at once (collective), depending on the context. The differences in the two interpretations can result in two different plans in a rule like Pick up all green blocks before picking up a red block. This problem is aggravated when multiple collections are involved. For example, Pick up the green blocks, the red blocks, and then the blue blocks could be interpreted in 2^3 different ways.

This ambiguity does not occur if the semantics of an action or predicate uniquely determines the meaning of a sentence. For example, John and Mary went to school is obviously distributive, since both John went to school and Mary went to school could be inferred. On the other hand, John and Mary met is obviously collective. Note that many actions or predicates which can be interpreted as either distributive or collective contain more than a single lexical item. For example, a singular individual can win a 100 meter dash, but only a group of people can win a relay race [5]. For our purposes, we will not be concerned with disambiguating the interpretation of a sentence containing collections; we will concentrate on properly representing the given interpretation.

Variable Dependency Collections often involve quantifiers. For example, All who left wore their sunglasses requires representations for the collection all who left that could be interpreted in two different ways as discussed. Without any contextual information, the predicate wear one's sunglasses is distributive as represented by $(\forall x)(Left(x) \rightarrow (\exists y)(Sunglasses(y) \wedge BelongTo(y, x) \wedge Wear(x, y)))$, where the universal quantifier dominates the existential quantifier. Therefore, the proposed representations of collections should be able to handle the proper dependency between variables.

Rules in Planning When a rule concerns collections, the concrete constituents of the collections are not known until the rule is fired. For example, in the rule Pick up a red block after picking up all green blocks on the table, green blocks on the table is a variable collection whose constituents vary according to the rule firing environment. These types of rules involving collections are often required in planning, so that variable collections as well as constant collections need to be represented.

4 Representations of Collections

4.1 Constant Collections

Two types of constant collections are defined below and corresponding representational schemata are shown.

Definition 1 *A* **hydra** *is a collection that is introduced by identifying all of its constituents.*

Figure 1 is a case frame for a hydra.

Definition 2 *An* **atomic collection** *is a collection that is introduced by a denoting phrase that does not specify all constituents.*

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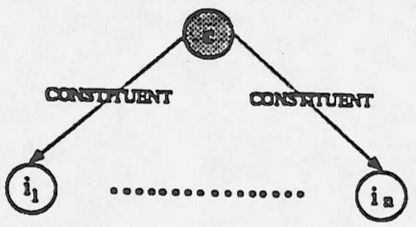
[4] An intensional set is a set without a corresponding extension, such as the set of unicorns.

SYNTAX:
If $i_1, \ldots, i_n$ ($n \geq 0$) are distinct individual nodes, and 'c' is an identifier not previously used, then



is a network, and c is a structured individual node.

SEMANTICS:
[c][§] is a collection, called a hydra, whose constituents are $[i_1], \ldots, [i_n]$.

Figure 1: Case Frame for Hydra

A denoting phrase can be decomposed into a number of denoting propositions and propositions describing known constituents. For example, *All who left including John* could be decomposed into a denoting proposition that represents the phrase *who left* and a proposition describing John as a constituent. Denoting propositions of an atomic collection are descriptions of the collection, and the syntax can be restricted as follows:

Definition 3 A denoting proposition [p] of an atomic collection c is a proposition, and the cable set [8] of p
(i) is {<COLLECTION,{c}>, <SIZE,{s}>}, where s is a numeral, or
(ii) is {<SUBCOLLECTION,{c}>, <SUPERCOLLECTION, {i}>}, where i is a base node or a hydra, or
(iii) is {<SUPERCOLLECTION,{c}>, <SUBCOLLECTION,{i}>}, where i is a base node or a hydra, or
(iv) contains a cable <COLLECTION,{c}>, whose variation <r,{i}>, with r ∈ {AGENT, OBJECT, OBJECT1, OBJECT2, ...} and an individual node i, is a cable of a proposition node in SNePS.

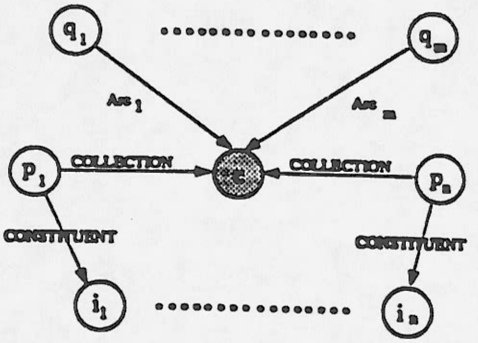
Figure 2 is a case frame for an atomic collection.

The possibility of two different interpretations of predicates involved in the descriptions of some collections requires two different types of representational schemata. This can be done by using different arc labels as described below.

Collective Interpretation On the collective reading, a predicate is applied to a collection, not spread over its constituents, and the collection is treated as one unit. COLLECTION-PROPERTY or COLLECTION-ACT arcs will be used to indicate that the collection as a whole is an argument of a predicate.

Distributive Interpretation On the distributive reading, a predicate is spread over the constituents of a collection, and a collection is treated as a cumulative entity composed of its constituents. COLLECTION-DPROPERTY or COLLECTION-DACT arcs will be used to indicate that every constituent of a collection is an argument of a predicate.

SYNTAX:
If $i_1, \ldots, i_n$ ($n \geq 0$) are distinct individual nodes, $p_1, \ldots, p_n$ are distinct proposition nodes, $q_1, \ldots, q_m$ ($m > 0$) are distinct denoting proposition nodes, and 'c' is an identifier not previously used, then



is a network and c is a base node, where $\mathrm{Arc}_j \in$ {COLLECTION, SUBCOLLECTION, SUPERCOLLECTION}, for $1 \leq j \leq m$.

SEMANTICS:
[c] is an atomic collection whose denoting propositions are $[q_1], \ldots, [q_m]$. $[i_1], \ldots, [i_n]$ are known constituents. $[p_j]$, for $1 \leq j \leq n$, is the proposition that $[i_j]$ is a constituent of [c].

Figure 2: Case Frame for Atomic Collection

SYNTAX:
If c is a base node and 'v' is an identifier not previously used, then

is a network and the v is a structured variable node.

SEMANTICS:
[v] is the typical constituent of an atomic collection [c].

Figure 3: Case Frame for Typical Constituent of a Collection

Typical Constituent A typical constituent is an arbitrary constituent of an atomic collection that is captured from descriptions of the atomic collection. When an atomic collection is introduced by the descriptions, the concept of its typical constituent may or may not be present in the descriptions. However, for complete representation, we need to provide a schema for the representation of a typical constituent of an atomic collection. In fact, there are cases where the concept of typical constituent is essential, as in *All who left wore their sunglasses* and *The youngest computer scientist* (See Figure 5 a, b). The typical constituent of an atomic collection should be a variable. Figure 3 is a case frame for a typical constituent. The notion of a typical constituent will also be used in Section 4.2.

4.2 Variable Collections

An atomic collection is independent of its constituents, whereas a collection in a rule is a tool to recognize specific

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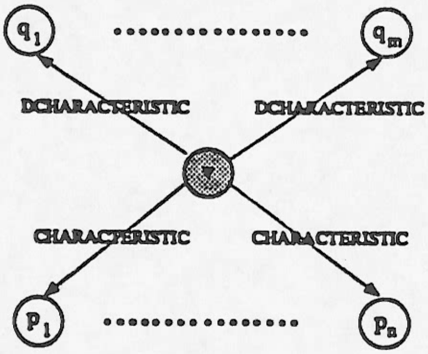
[§] For any node n, [n] is the meaning of n. [8]

SYNTAX:
If $p_1,....,p_n$ ($n >= 0$) and $q_1,....,q_m$ ($m >= 0$) are distinct proposition nodes, where $n + m > 0$, and 'v' is an identifyer not previously used, then



is a network and v is a structured variable node. All the p_i and q_j, $1 \leq n$, $1 \leq m$, dominate v.

SEMANTICS:
$[\![v]\!]$ is a variable collection whose collective characteristics are $[\![p_1]\!],....,[\![p_n]\!]$, and distributive characteristics are $[\![q_1]\!],....,[\![q_m]\!]$.

Figure 4: Case Frame for Variable Collection

constituents according to the environment in which the rule is fired. Figure 4 is a case frame for a variable collection. A rule like When there are more than one plans, find the best plan requires representing the typical constituent, since the best constituent of a collection has to be represented (See Figure 5 (b)). When characteristics of a variable collection require representation of the typical constituent of the collection, the case frame for typical constituent (Figure 3) can be utilized.

4.3 Examples

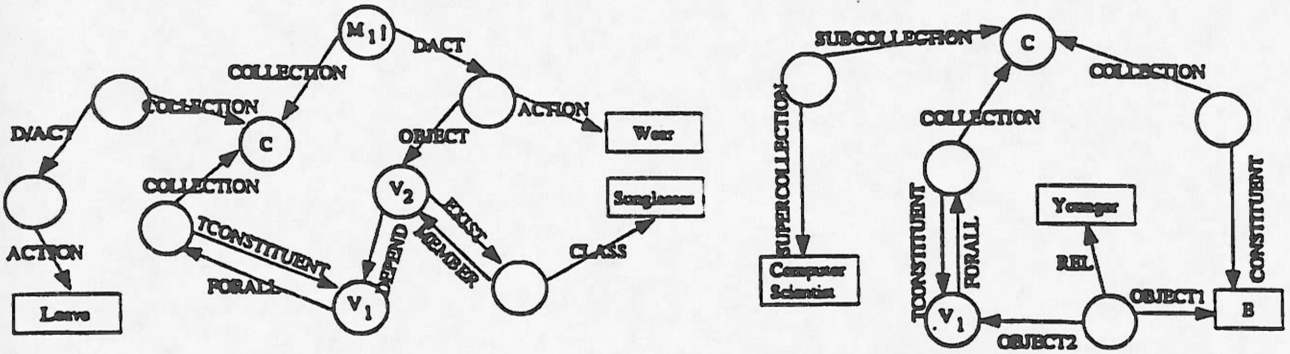
The case frames defined above are capable of representing many widely different instances of collections as illustrated in Figure 5. Specifically, it can represent:

1. distributive and collective interpretations of sentences with collections (Figure 5 a, c, d, and e).
2. the size of a collection, if given (Figure 5 e).
3. a hydra (Figure 5 c).
4. an atomic collection with distributive characteristics (Figure 5 a, c, and e).
5. an atomic collection with collective characteristics (Figure 5 d).
6. an atomic collection with distributive characteristics and collective characteristics (Figure 5 f).
7. an atomic collection with its typical constituent (Figure 5 a, b, d, and f).
8. the best constituent of a collection when a comparison method is available (Figure 5 b).
9. two collections with the same constituents (Figure 6 a).
10. variable collections (Figure 7).

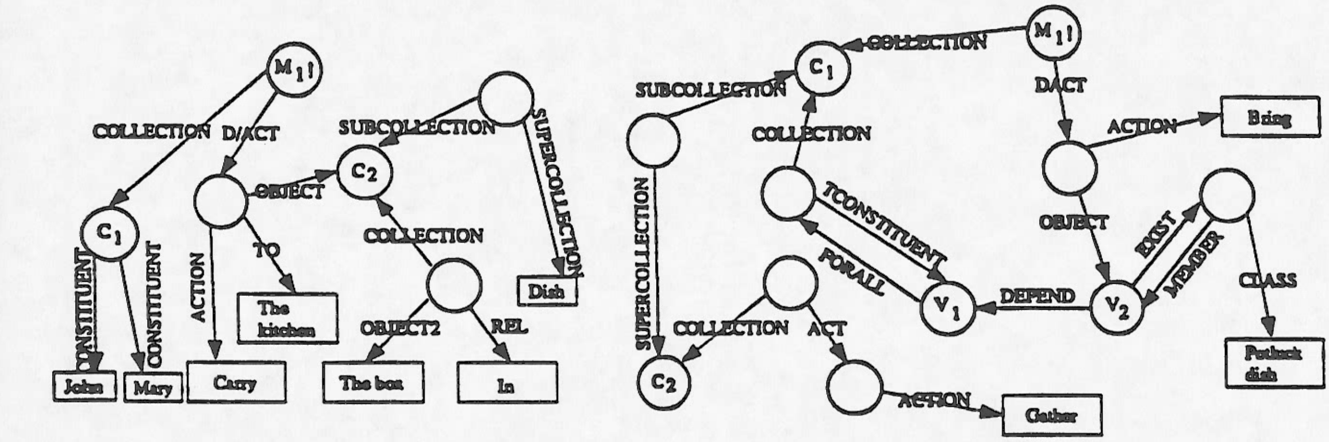
Group A group could be represented as an atomic collection with a proper name. For example, in John and Mary are *committee A*, *committee A* is an atomic collection with a proper name, and John and Mary is a hydra that plays the role of committee A. It is possible that two different groups have the same participants:

> John and Mary are *committee A*.
> John and Mary are *committee B*.
> *Committee A* went to Africa.

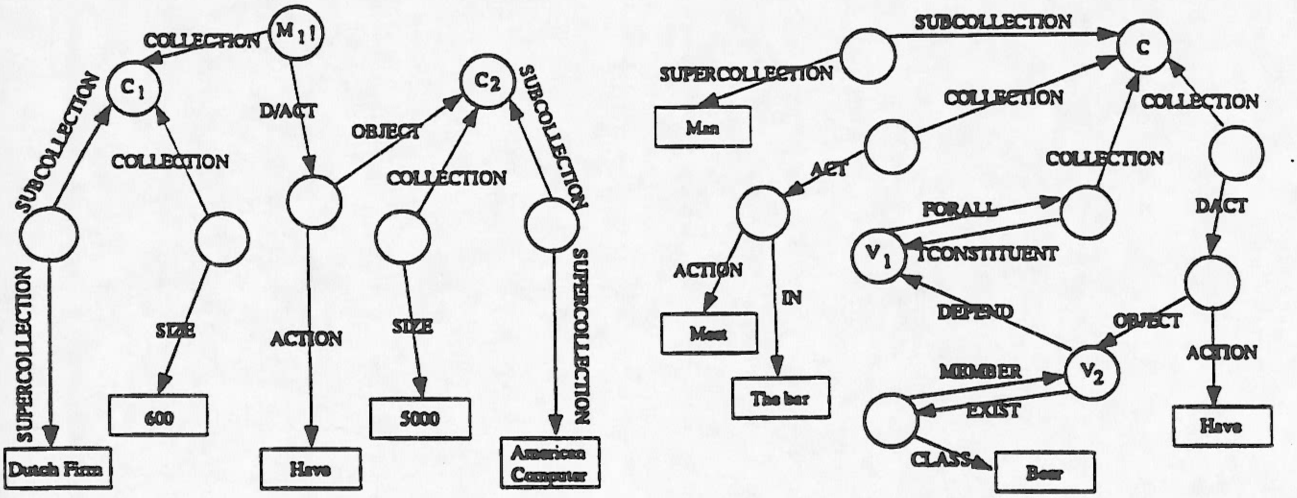
John and Mary went to Africa can be inferred from the above context; however, *Committee B* went to Africa can not. Figure 6 b shows the representation of a group.



(a) All who left wore their sunglasses. (b) The youngest computer scientist.

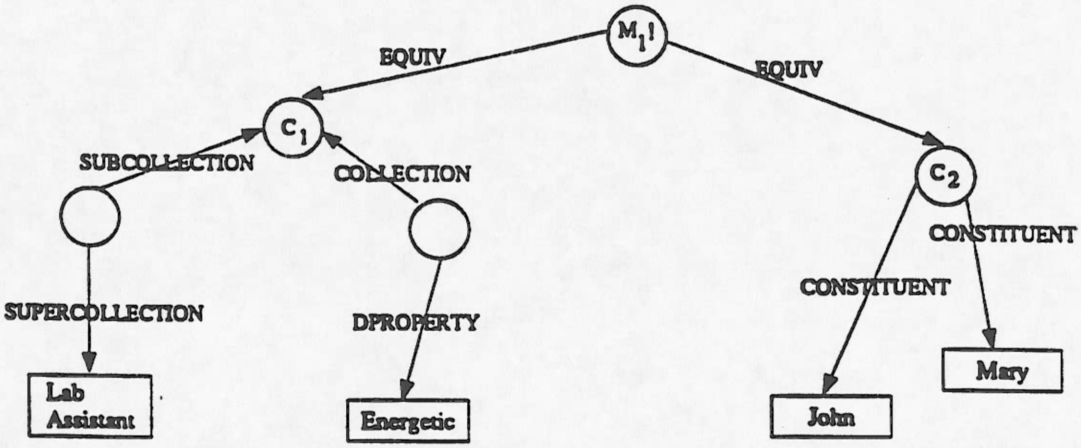


(c) John and Mary carried the dishes in the box to the kitchen. (d) Some of those who gathered brought potluck dishes.

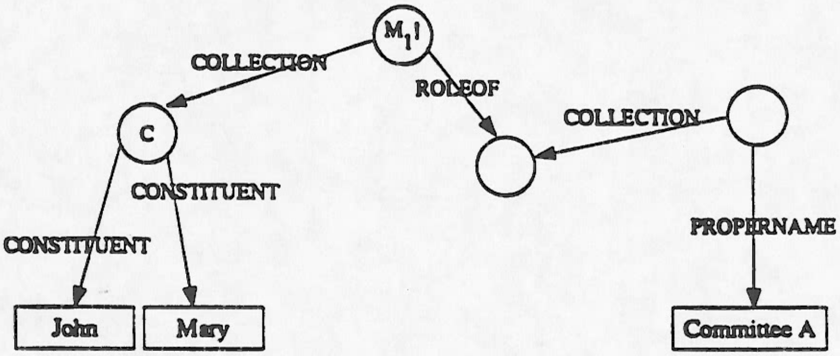


(e) 600 Dutch firms have 5000 American computers. (f) The men who met in the bar while having a beer.

Figure 5: Representations of Propositions with Constant Collections

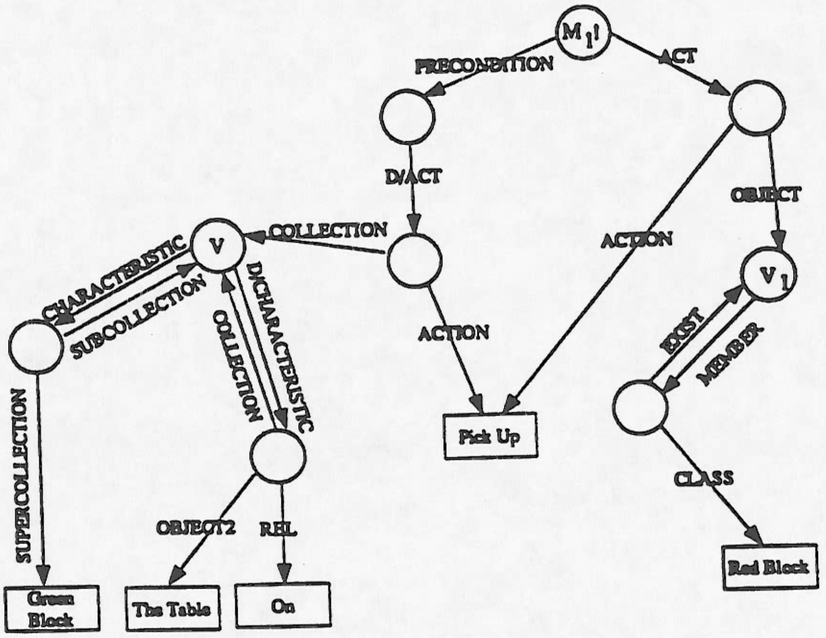


(a) The energetic lab assistants were John and Mary.

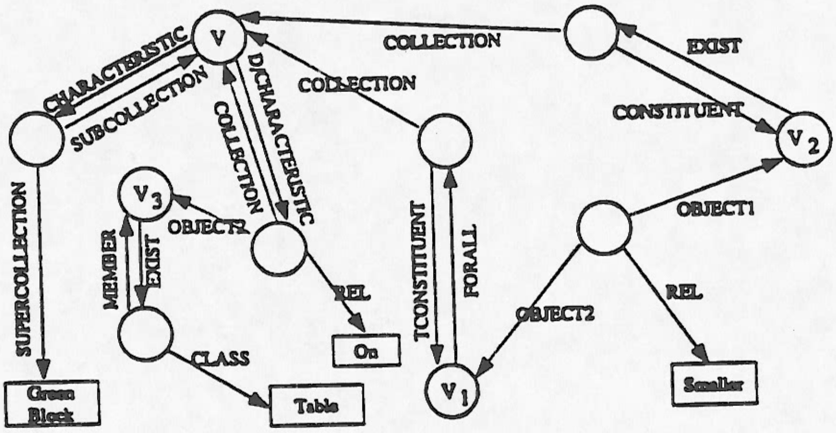


(b) John and Mary are Committee A.

Figure 6: Representations of a Coreferential Collection (a) and a Group (b)



(a) Pick up a red block after picking up all green blocks on the table.



(b) The smallest green block on a table.

Figure 7: Representation of a Rule with a Collection

5 Conclusions

We presented SNePS case frames for intensional representation of various types of collections including constant collections and variable collections. A schema for representing a typical constituent of a collection was also defined. Applications of these frames to representation of a wide variety of propositions involving collections were demonstrated. We also showed how different interpretations of sentences with collections as well as collection sizes can be represented using the proposed schemata.

Further work in this area concerns many important issues on reasoning about collections based on the proposed representations. There should be a sound counting mechanism for solving problems closely related to the classical problem of handling cardinality in fully intensional knowledge representation [1, 4]. Properties of collections need to be inherited in many occasions. Individual constituents of a collection should inherit distributive characteristics of the collection. Similarly, a subcollection would share descriptions about its supercollections. In order to instantiate a variable collection, unification mechanisms need to be developed. Operators analogous to set union, intersection, difference need to be developed, as well as mechanisms to take care of additions or deletions of a constituent. The counting mechanism should be extended to be able to keep correct counts after these operations, if relevant.

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