Now: Between Indexical Thoughts and Indexical Thought

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Abstract. It has long been argued that rational action necessitates having indexical thoughts. The ability to understand and generate indexical expressions is also a precondition for interacting in natural language. A rational acting agent that interacts in natural language is, thus, required to develop a proper treatment of indexicality. Treatments of indexicality in the literature have, almost always, assumed an indexical language of thought. I shall argue that, in most common cases, all indexical expressions in said language may be reduced to expressions involving only counterparts of "I" and "now". I dismiss the language-of-thought "I" as being not indexical at all; this leaves us with "now". First, I review past approaches to representing and reasoning about "now", and systematically evaluate them against four criteria which I develop and motivate. Second, I push forward a totally different treatment of "now", based on a grounded, layered agent architecture. In this architecture, the language of thought—at the top, knowledge layer—is a classical, first-order, non-indexical language; the indexical "now" features at a lower, perceptuo-motor layer. It is the reasoning process that cuts across both layers, giving rise to indexical thinking. Thus, we trade *indexical thoughts* for *indexical thought*. The proposed treatment is shown to supersede previous approaches with respect to the proposed criteria.

1 INTRODUCTION

Nothing is perhaps more common to our talk than indexical expressions. Indexicals, such as "I", "you", "this", "here", and "now" are familiar to native speakers and provide quite efficient means of communication. Their distinguishing feature is that, though unambiguous, their reference is determined and often *only* determined by the non-linguistic context of use. Thus, upon hearing an indexical like "I", one immediately identifies the speaker as its referent; no knowledge other than knowledge of the context of utterance is required. On the other hand, non-indexical expressions typically mandate knowledge of the linguistic context (for example, with pronouns) or general world knowledge (for example, with proper nouns or descriptive noun phrases) to identify their referents. Yet, indexicals have proved to be tough beasts to the semanticist and the philosopher, as attested by the amount of ink they have drawn over the years [13, 30, 20, 3, 22, 5, for example].

According to Kaplan's classical treatment of indexicals [13], the interpretation of linguistic expressions happens in two stages (three, if we consider intensionality): given an expression, we, first, consider its *character*, and, second, given the context, we compute the *content* of the expression. Here, the content is the referent of the expression, and the character is a function from contexts to contents. For non-indexical expressions, the character is a constant function. For indexicals, the character determines the *meaning* of the expression. For example, the characters of "I", "now", and "here" map a context to the speaker, the time, and the location of the context, respectively. Kaplan's theory did not go unchallenged, and several authors have attacked several aspects of it [30, 20, 3, for example]. Although I will have nothing much to say about these debates, I will come back to them later.

The aspect of indexicals that I am concerned with in this short note has to do with the observed relation between rational action and indexicals. In a (rightly) celebrated article [21], John Perry argues that beliefs underlying rational action often involve an essential indexical—one that cannot be replaced by a non-indexical or, by any means, be swept under the carpet.[2] But what is an indexical thought? And why is it necessary for rational action? The following examples will illustrate the point.

The Case of the Messy Shopper [21]. You are in the supermarket and you notice a trail of sugar on the supermarket floor. You go around, following that trail, trying to find the messy shopper. Suddenly, you realize that *you* are the messy shopper and, hence, you adjust your own sac of sugar.

What is it that you came to believe which motivated you to adjust your sac of sugar?

The Case of the Lost Hiker [21]. You are hiking and you get lost. You have a map, and you reason that if you can get to a certain, marked location on the map, Mt. Tallac, you will be able to get back on track. After some aimless walking, you suddenly realize that you are facing Mt. Tallac. Accordingly, you start following the map to your destination.

What is it that you came to believe which motivated you to start following the map?

The Case of the Fire Alarm [11]. You believe that whenever the fire alarm sounds you should leave the building. Suddenly, you hear the fire alarm and, hence, you leave the building.

What is it that you came to believe which motivated you to leave the building?

According to most authors (particularly, Perry), the answers to the three questions is that it is an indexical thought that triggered the reasoning agent to act. In particular, these are the beliefs "*I* am the messy shopper", "Mt. Tallac is *here*", and "the alarm is sounding *now*", respectively. As Perry suggests, the indexicals in these thoughts are essential since they cannot be replaced by any co-referring terms. Thus, coming to believe that "John Perry is the messy shopper" or "the author of 'the problem of the essential indexical' is the messy

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[2] However, among others, Millikan [17] argues that such beliefs indeed include certain essential ingredients that are, nonetheless, not indexical.

shopper" cannot possibly be what would motivate John Perry to adjust his sac of sugar, since he might not believe that *he* is John Perry or that *he* is the author of 'the problem of the essential indexical.' Similarly, coming to believe that "the alarm is sounding at 12:00" at 12:00 does not explain why the agent would leave the building unless it also believes that "now is 12:00." Ditto for the lost hiker.

In artificial intelligence (AI), since acting agents are our favorite subjects of research, we have to face Perry's problem head on. Approaches to indexicality in AI, almost unanimously, take Perry's remarks about "indexical thoughts" pretty faithfully, by building indexicality in the logical language used for representing the reasoning agent's beliefs [6, 15, 16, 1, 4]. I beg to differ. Although I agree with Perry, and with those authors, that indexicality is essential for rational action, I contend (partly with Millikan in [17]) that indexicality is more suitably built, not in the knowledge representation language, but in the interaction of reasoning and acting. Thus, I propose to trade indexical *thoughts* for indexical *thought*.

Here is my plan. In Section 2, I adopt and motivate the position that for the purpose of rational action and straightforward linguistic interaction, all relevant indexicals are reducible to "I" and "now." Hence, I argue that, under some reasonable assumptions, "I" is not indexical at all. Thus, a proper treatment of "now" is all that is indeed for a proper treatment of indexicality (for rational action and simple linguistic interaction). In Section 3, I lay down some criteria for evaluating such a treatment and I evaluate previous approaches against them. In Section 4, I present my own proposal, evaluating it with respect to our criteria in Section 5.

2 "I" and "Now"

Kaplan assumes that contexts of utterance satisfy the requirement that the speaker is at the location of the utterance, at the time of the utterance [13]. Hence, he concludes, all utterances of "I am here now" are analytically true. This conclusion came under fire by several authors and for several reasons. For example, certain uses of indexicals in linguistic discourse that are common, but not typical, involve so-called "deferred reference":

(1) Condemned prisoner: "I am traditionally allowed to order whatever I like for my last meal" [20, p.20]

(2) Chess instructor to student: "According to all textbooks, you often get in trouble with that move" [20, p.21]

(3) Medical pathologist pointing at a spot on his chest: "When a person is shot here, we can usually conclude that it was not suicide" [20, p.29]

In addition, occurrences of indexical expressions in written or recorded messages are also problematic for Kaplan's theory.

(4) Message played by an answering machine: "I am not here now" [30]

Had we been concerned with fully interpreting linguistic utterances and generating subtle ones, we will have had to face such examples head on. But since our main concern is the role of indexicality in rational action and straightforward linguistic discourse, we may safely disregard such uses of indexicals. In particular, I assume that knowledge representation and reasoning are carried by and in a logical language of thought. There is no evidence, and there is no reason why we should assume, that expressions in said language of thought include similar uses of indexicals.[3] As pointed out, the above data will be pressing if we are to tackle the tricky issue of translation between the language of thought and natural language.

Hence, I will assume that, in a language of thought, all indexicality may be reduced to expressions involving only "I" and "now", as illustrated in Table 1.

Expression	Reduction
here	the location of I now
you	the addressee of I now
(demonstrative) this	the object you is pointing at now
today	the day of now
tomorrow	the day after today

Table 1. Indexicals in terms of "I" and "now"

Of course the entries in the above table do not account for all indexical expressions, but they include those which, it is reasonable to assume, play a role in rational action (especially, "you", "here", and "this").

Now, are "I" and "now" really indexical? I think it is beyond doubt that they, or their counterparts in other natural languages, are indeed indexical. However, remember that we are not concerned with natural languages. Rather, our focus is on a language of thought (LOT). I believe that the LOT "now" is indexical, but the LOT "I" is not. To see why, note that there are at least two differences between a LOT and a natural language:

1. a LOT is private to each agent, a natural language is public; and
2. natural language expressions can be uttered, LOT expressions cannot be uttered, they can only be thought.

Alluding to Kaplan, indexicality of LOT is, thus, rooted in sensitivity to a context of thought, not a context of utterance. Hence, the character of LOT "I" is the *thinker* of the context. But, since the LOT is private, this character is a constant function, yielding the same thinker in every context, namely the reasoning agent whose LOT we are considering. Hence, nothing is indexical about LOT "I." It is just a constant of the LOT, which is psychologically marked for action.[4]

LOT "now", on the other hand, retains its indexicality. Since thought occurs over time, two numerically distinct contexts of thought necessarily have different times. Thus, the character of LOT "now" is not a constant function and, hence, LOT "now" is indeed indexical.

3 "Now"

3.1 Tense-Marker and Post-It-Note "Now"

We may distinguish two uses of the English "now" that correspond to two functions LOT "now" may serve. These two uses are exemplified by the following sentences.

(5) Speaker looking outside a window: "It is now raining."

(6) A note posted on a door: "I am not in now."

[3] Millikan [18] might disagree though.
[4] How are we then to explain the behavior of the messy shopper? Wait for Section 5.

In (5), "now" merely serve as a tense marker, indicating that the raining is contemporaneous with the utterance. Let us to refer to such uses of "now" as *tense-marker* "now." In (6), "now" is used to refer, not to the unique time of writing the note, but to any time at which the note is read [22, 5]. Thus, the "now" written on the post-it note changes its reference as times goes by. Let us refer to such uses of "now" as *post-it-note* "now."[5]

These two uses of "now" serve quite important purposes in a LOT. Tense-marker LOT "now" allows an agent to distinguish the present from the past and the future. There are at least two reasons why this distinction is important. First, if the agent expresses its beliefs in natural language, then a distinction between the present, the past, and the future is required for generating sentences with the correct tense expressing what has happened, is happening, or will happen. Second, and more important, present facts have a distinguished status for an acting agent. For example, for our fire alarm agent from Section 1, if we tell the agent that the alarm sounded yesterday, it merely needs to remember this fact, and maybe derive some inferences from it. However, if we tell it that the fire-alarm is *now* sounding, it also needs to act on this fact and leave the building.

Instead of posting a note on a door, acting agents often use mental post-it notes to remind themselves of important future actions. Again, for our fire alarm agent, forming the attitude that whenever the alarm sounds it should leave the building may be achieved by posting a mental note saying (in mentalese) "if the fire alarm is now sounding, leave the building." The LOT "now" occurring in this LOT expression is a post-it-note "now", not a tense-marker "now."

While it may seem that tense-marker "now" is more common than the exotic post-it-note "now", it is interesting to note that most approaches to the representation of "now" in AI have been about the latter and not the former. We need to investigate these approaches; but, first, we should consider how we are to evaluate them.

3.2 Criteria for an Adequate Treatment

For a treatment of "now" to be adequate for a rational agent that can potentially interact in simple natural language (cf. [26]), reporting on what it has done and is doing, it has to satisfy at least the following four criteria (listed in no significant order).

N1. *The treatment should account for tense-marker "now" and post-it-note "now."* A treatment which accounts for one and not the other is lacking.

N2. *The treatment should account for temporal progression.* As pointed out above, LOT "now" is always changing its referent. A treatment of "now" which does not reflect this intuition is (i) not psychologically adequate and (ii) cannot accommodate an acting agent, since acting does take time.

N3. *The treatment should be as computationally tractable as possible.* Of course, this is a matter of degree, but nothing is more mundane than the passage of time, and a treatment that burdens the agent with a relatively heavy computation to catch up with the passage of time is both psychologically and computationally unfavorable.

N4. *The treatment should not make unmotivated assumptions.* In general, a treatment should not impose constraints on, e.g., the structure of time, the agent's beliefs, the agent's actions, that are only needed for the treatment to be adequate. Again, this is a matter of degree, but assumptions about the agent or the ontology should be independently motivated as much as possible.

Given the above criteria, let us consider how existing treatments of "now" fair.

3.3 "Now" in the Literature

Treatments of "now" in the literature may be divided into three major classes. First, there are the classical Priorian tense logics [23]. Classical tense logic is essentially a temporally-interpreted bimodal logic. If p is a proposition, "Pp" means that "It has been the case that p" and "Fp" means that "It will be the case that p". By itself, "p" refers to the *current* truth of p. Thus, syntactically, the present is distinguished by having the proposition outside the scope of any tense operators. Semantically, expressions (which may be embedded within tense operators) are interpreted with respect to a particular temporal index representing the present. Other treatments, within the same framework, explicitly introduce a "now" tense operator N to account for the curious property of the English "now" that, in typical uses, it always refers to the time of utterance even when embedded within a nest of tense operators [12].

Although it clearly accounts for tense-marker "now", the tense logical approach fails to account for post-it-note "now" and for temporal progression; thus violating **N1** and **N2** and, hence, avoiding **N3** and **N4**.

The second approach, usually adopted in reasoning about actions and plans, is to represent the present using an indexical **now** term. The use of indexical terms, in general, was studied in depth by Lespérance and Levesque in [16] with special attention to the case of **now** in [15].[6] The indexicality of such a term stems from its having a context-dependent interpretation, much in the same spirit of Kaplan's semantics discussed above. However, unlike the English "now", whose content depends on the context of utterance (or assertion), the semantics of the indexical **now** depends on the *evaluation* context. In the context of acting and planning, it is the time of executing a particular instance of a plan that includes occurrences of **now** in its specification. Along the lines of [15] (and using the same syntax), the following is a possible representation of a plan to get to other side of the street (probably for a rather despondent agent):

$$\mathbf{if}(\mathbf{At}(\mathbf{now}, \textsc{WalkLightOn}), \textsc{Cross}, \mathbf{noOp})$$

This roughly says that, if, at that the time of performing the action, the walk-light is on, then cross the street; otherwise do nothing. What should be noted is that **now** in the above form does not refer to the time of introducing the form into the knowledge base, or to any other *fixed* time for that matter. It is, in a sense, a place-holder for any time at which the plan is performed.

What about temporal progression? Lespérance and Levesque briefly discuss an approach which we will now consider in some detail. The obvious approach to modelling the passage of time within the theory of [16] would be to appropriately edit the knowledge base every time "now" changes in order to preserve the truth of its sentences. Thus, **At**(**now**, RAINING) should be replaced by something more appropriate once "now" changes. One problem, of course, is that such updates are computationally expensive. To get around the problem, [16, p. 101] suggest that "if all occurrences of 'now' are replaced by a new constant and the fact that this new constant is equal

[5] It should be clear that nothing is special here about the word "now" itself; "now" may be dropped from both (5) and (6) without affecting their propositional contents. Rather, it is the present tense in both sentences that really plays the two indicated roles.

[6] Other authors have also used the same or a similar approach [25, 1, 4].

to 'now' is added, then only this single assertion need be updated as time passes." This indeed eliminates the problem of expensive belief update and provides a neat logical and computational account of "now".

I believe that Lespérance and Levesque's treatment of "now" satisfies **N1** and **N2**. However, I also believe that it does not fair well with respect to **N3** and **N4**.

First, note that, though they do not mention it, Lespérance and Levesque's treatment of equality will have to be subtler than usual, if their approach to temporal progression is to work effectively. For example, one should block instances of rules like demodulation when the term involved in the conclusion is **now**, since we do not want to express any transient beliefs using **now** itself but, rather, the non-indexical term it is currently equal to. This is a violation of both **N3** and **N4**.

Second, from a cognitive perspective, I find the very idea of *erasing* sentences from an agent's knowledge base as time passes by far from natural. If such sentences represent beliefs that the agent once held, where do they go, and how come the agent would have no memory of them once time passes? Note that this cannot be explained away as a matter of forgetting, for forgetting is not that selective to always affect beliefs involving "now", nor is it vigorous enough to take effect with every tick of the clock. The only way to explain this mysterious disappearance of beliefs is by arguing that they exist at a lower level of consciousness with respect to other beliefs. If this were the case, why are such beliefs part of the logical theory (which we take to be representing conscious beliefs of the agent)? This points to a possible violation of **N4**.

The third approach to represent "now" is to do it indirectly, by means of a *Now* predicate, where the expression $Now(i)$ means that the current time is represented by the term *i*. This is exactly the method adopted in *active logic*, originally known as *step logic* [6]. Temporal individuals are represented by integers, with the usual numerical order implicitly representing chronological order. In active logic, time moves with every inference step. This movement of time is represented both logically and meta-logically. Logically, this is achieved by a special inference rule that essentially replaces $Now(i)$ by $Now(i{+}1)$. Meta-logically, assertions are associated with the step, i, of inference at which they were asserted.

Though there clearly is an account for tense-marker "now" in active logic, I am not aware of an explicit treatment of post-it-note "now." **N2** and **N3** are, I believe, observed by active logic. However, it is **N4** which I think is somehow violated. Apparently, the use of integers facilitates the expression of some crucial rules of inference (also the *counting* of reasoning steps [19]) that depend on having a well-defined notion of the *next moment of time*, represented by the integer successor operator. However, such a representation forces a certain degree of rigidity on the kind of knowledge that may be entered into the system. For example, there is no way to assert at step $i{+}m$ $(m > 1)$ that a certain event e_2 occurred between events e_1 and e_3 that happened at times i and $i + 1$, respectively. In other words, once "now" moves, there is no way to go back and create arbitrary past temporal locations. This is definitely a big drawback if the system is to be used in interacting with humans, where assertions need not be only about the present.

4 INDEXICAL REASONING WITHOUT INDEXICAL REPRESENTATION

From the foregoing discussion, we may conclude that some way of representing and reasoning about "now" is essential for rational action. Moreover, it seems fair to conjecture, at least temporarily, that it is *only* essential for rational action. That is, it is action—and not merely armchair reasoning *about* action, but actual act*ing*—that mandates a treatment of indexicality; in the absence of action, no indexical is essential. This conjecture cannot be fully defended at this point; suffice it to say that all debates about indexicals are about the role they may or may not play in explaining behavior [21, 17, 31, for example]—no one has ever tried to argue for indexical thoughts in the absence of action.[7] Even if, in the final analysis, indexicals turn out to have a more prominent role, independent of action (although I cannot imagine how), the treatment of "now" that I shall outline may still be valuable in relating indexicals to action.

But how can we relate indexical reasoning to action without mentioning a framework for relating the two activities: reasoning and acting? The work cited in Section 3, presents logical frameworks, without a mention of how the symbols of the logical language are grounded, nor of how the reasoning activities guide action. These issues are implicitly assumed to be somehow treated and they, no doubt, are. It is my conviction, however, that said treatment may be at least useful to consider as part of the very approach to indexicality. Hence, I will present an approach to reasoning about "now" based on a grounded agent architecture—GLAIR.

4.1 GLAIR

In the sequel, I assume a theory of agents based on the GLAIR agent architecture [7, 26]. GLAIR is a layered architecture consisting of three layers:

1. **The Knowledge Layer (KL):** The layer at which symbolic reasoning takes place. This layer may be implemented in any logic-based system, where anything that we may think or talk about is abstractly represented by a term, including actions and behaviors. (Historically, the KL has been implemented by the SNePS knowledge representation, reasoning, and acting system [27, 28, 29].) This is also the level responsible for interpreting composite action terms, and scheduling them for execution, as a result of reasoning or natural language instructions [26].
2. **The Perceptuo-Motor Level (PML):** The layer at which routines for carrying out primitive acts are located. This layer also includes an elaborate representation of perceivable physical objects, properties, and relations, typically in terms of feature vectors resulting from the processing of sensory input. The representation of an entity at this level is more fine-grained than the symbolic representation at the KL, so that the agent may, for example, be able to perceptually distinguish two physical objects given their PML representations, though it may not be able to discern them by mere KL reasoning. KL terms that represent objects which are also represented at the PML are grounded in the corresponding PML representations, through a relation of alignment. Likewise, KL terms representing primitive actions are aligned with the corresponding PML routines.
3. **The Sensori-Actuator Level (SAL):** The layer controlling the operation of sensors and actuators. I will have nothing much to say about the SAL henceforth.

The treatment of "now" to be presented below is based on the intuition that recognizing the passage of time is more a process of per-

[7] In Lespérance and Levesque's argument for indexical "reasoning" [15], it is actually not the "reasoning" per se that is required to be indexical. Rather, it is the representation of a plan that, they claim, mandates indexicality for its correct execution.

ception than one of conscious reasoning; one does not need to reason in order to determine that time has passed, one just *feels* the passage of time. Hence, tense-marker "now" will be accounted for by a careful synchronization of PML and KL activities which, I claim, gives rise to the sense of temporal progression. Post-it note "now", on the other hand, is accounted for by building temporality in the very process of *practical* reasoning which, unlike armchair reasoning, is not limited to the manipulation of KL terms, but also involves consulting PML structures.

4.2 Language

I will take the KL language to be a first-order, sorted language $\mathcal{L}$, intended to be the language of thought of the agent. In what follows, we identify a sort s with the set of symbols of sort s. $\mathcal{L}$-terms are partitioned into eight base syntactic sorts, σ_A, σ_G, σ_E, σ_T, σ_S, σ_N, σ_C and σ_O. Intuitively, terms of each sort respectively denote actions, agents, events, times, propositional fluents (or "states"), names, clock readings, and objects. Each denoting symbol of $\mathcal{L}$ belongs to a unique sort from a set Σ of syntactic sorts. The set Σ is the smallest superset of $\sigma = \{\sigma_A, \sigma_G, \sigma_E, \sigma_T, \sigma_S, \sigma_N, \sigma_C, \sigma_O\}$ containing the following sorts.

1. $\bigtimes_{i=1}^{k} \tau_i$, and
2. $(\bigtimes_{i=1}^{k} \tau_i) \longrightarrow \tau$

where $\tau_i, \tau \in \sigma$, for $1 \leq i \leq k$, for every $k \in \mathbb{N}$. Intuitively, the above accounts for the syntactic sorts of k-adic predicate and function symbols, respectively.

The alphabet of $\mathcal{L}$ is made up of Boolean connectives ($\neg, \wedge, \vee, \supset, \equiv$) and quantifiers ($\forall, \exists$); a set of syncategorematic punctuation symbols; a countably infinite set of variables; a set of domain-dependent constants, function symbols, and predicate symbols; and the two special symbols When and Whenever.

As usual, terms of $\mathcal{L}$ are defined in the standard way as the closure of the set of constants and variables under combination with function symbols, provided that sort restrictions are observed. Similarly, well-formed formulas (WFFs) are defined as in any sorted first-order language. In addition to terms and WFFs, $\mathcal{L}$ includes another kind of expression—well-formed directives (WFDs). A well-formed directive is an expression of the form $\mathsf{Whenever}(s, a)$ or $\mathsf{When}(s, a)$, where $s \in \sigma_S$ and $a \in \sigma_A$. Directives provide the link between reasoning and acting [14] and are akin to Millikan's pushme-pullya representations [18]. Intuitively, $\mathsf{Whenever}(s, a)$ means that the agent will attempt to execute the action a whenever it comes to believe that s holds. $\mathsf{When}(s, a)$ is a once-only variant, where the agent follows the directive only once.

A full, careful exposition of the semantics of the WFD-free fragment of $\mathcal{L}$ is not possible given space limitations and is, fortunately, mostly orthogonal to the issues at stake here. The unifying semantics of [2] more than suffices for the WFD-free fragment, but I briefly sketch important ingredients of $\mathcal{L}$-semantics for completeness. Expressions are interpreted with respect to an ontologically rich structure:

$$\mathfrak{M} = \langle \mathfrak{D}, < \rangle$$

where $\{\mathfrak{A}, \mathfrak{E}, \mathfrak{S}, \mathfrak{T}, \mathfrak{G}, \mathfrak{N}, \mathfrak{C}, \mathfrak{O}\}$ is a partition of the domain $\mathfrak{D}$. Intuitively, the parts are non-empty sets of, respectively, actions, events, states, time points, agents, names, clock readings, and objects. $<\subseteq \mathfrak{T}^2 \cup \mathfrak{C}^2$ is an irreflexive order which is partial on $\mathfrak{T}$ and total on $\mathfrak{C}$. The interpretation function $[\![\cdot]\!]^{\mathfrak{M}}$ with respect to $\mathfrak{M}$ is such that if e is a term of some base sort σ, then $[\![e]\!]^{\mathfrak{M}}$ is in the corresponding part of $\mathfrak{D}$. For convenience, the superscript $\mathfrak{M}$ will be dropped when there is no ambiguity.

Given the intended use of $\mathcal{L}$ to serve as the language of thought of an acting agent, we constrain $\mathcal{L}$ thus:

1. There is a constant $\mathsf{I} \in \sigma_G$ denoting the agent itself (for itself). There is a predicate symbol $\mathsf{Name} \in \sigma_G \times \sigma_N$, associating names with agents. $[\![\mathsf{Name}]\!]$ is neither total nor tabular at $\mathfrak{G}$ and $\mathfrak{N}$: agents do not necessarily have names and, if they do, said names need be neither unique nor exclusive, and names need not be associated with any agents.
2. Similar to the predicate Name, there is a function symbol $\mathsf{Clk} \in \sigma_C \longrightarrow \sigma_S$, where $[\![\mathsf{Clk}]\!] : \mathfrak{C} \longrightarrow \mathfrak{S}$ is a total, injective function, mapping each clock reading to the state of the clock's displaying it.
3. Constants of sort σ_T are countably infinite and $\mathfrak{T}$ is (possibly uncountably) infinite. Temporal order is represented by $\prec \in \sigma_T \times \sigma_T$, where $[\![\prec]\!] = <_{\mathfrak{T}^2}$ (the restriction of $<$ to $\mathfrak{T}^2$).
4. A predicate symbol $\mathsf{HoldsAt} \in \sigma_S \times \sigma_T$ represents the incidence of states on time points. $\mathsf{HoldsOn} \in \sigma_S \times \sigma_T \times \sigma_T$ represents *homogeneous* incidence on intervals: $[\![\mathsf{HoldsOn}]\!] = \{\langle s, t_1, t_2 \rangle |$ for every $t_1 < t < t_2$, $\langle s, t \rangle \in [\![\mathsf{HoldsAt}]\!]\}$. HoldsAt and Clk are synchronized in the following sense: if $\langle [\![\mathsf{Clk}]\!](c_1), t_1 \rangle, \langle [\![\mathsf{Clk}]\!](c_2), t_2 \rangle \in [\![\mathsf{HoldsAt}]\!]$, then $c_1 < c_2$ if and only if $t_1 < t_2$.
5. Event terms are constructed by a function symbol $\mathsf{Does} \in \sigma_G \times \sigma_A \longrightarrow \sigma_E$, where $[\![\mathsf{Does}]\!] : \mathfrak{G} \times \mathfrak{A} \longrightarrow \mathfrak{E}$ is a bijection. Hence, actions are the only events, and no group actions are considered. A predicate symbol $\mathsf{Occurs} \in \sigma_E \times \sigma_T \times \sigma_T$ represents event occurrence. Intuitively, $\langle e, t_1, t_2 \rangle \in [\![\mathsf{Occurs}]\!]$ when event e occurs on the interval (t_1, t_2).
6. Actions are atomic or composite. Atomic actions are denoted by functional terms formed by domain-dependent symbols and the special action term NoOp. Composite actions are, as usual, denoted by functional terms corresponding to imperative programming constructs: $\mathsf{Seq} \in \sigma_A \times \sigma_A \longrightarrow \sigma_A$, $\mathsf{If} \in \sigma_S \times \sigma_A \times \sigma_A \longrightarrow \sigma_A$, and $\mathsf{While} \in \sigma_S \times \sigma_A \times \longrightarrow \sigma_A$. The semantics is given in terms of constraints on action occurrences:
   - For every $a \in \mathfrak{A}$ and $\langle [\![\mathsf{Does}]\!](a, [\![\mathsf{NoOp}]\!]), t_1, t_2 \rangle \in [\![\mathsf{Occurs}]\!]$, $t_1 = t_2$.
   - For every $a \in \mathfrak{A}$ and $\langle [\![\mathsf{Does}]\!](a, [\![\mathsf{Seq}]\!](\alpha, \beta)), t_1, t_2 \rangle \in [\![\mathsf{Occurs}]\!]$, there is $t \in \mathfrak{T}$, $t_1 < t < t_2$ with $\langle [\![\mathsf{Does}]\!](a, \alpha), t_1, t \rangle, \langle [\![\mathsf{Does}]\!](a, \beta), t, t_2 \rangle \in [\![\mathsf{Occurs}]\!]$.
   - For every $a \in \mathfrak{A}$ and $\langle [\![\mathsf{Does}]\!](a, [\![\mathsf{If}]\!](s, \alpha, \beta)), t_1, t_2 \rangle \in [\![\mathsf{Occurs}]\!]$, if $\langle s, t_1 \rangle \in [\![\mathsf{HoldsAt}]\!]$, then $\langle [\![\mathsf{Does}]\!](a, \alpha), t_1, t_2 \rangle \in [\![\mathsf{Occurs}]\!]$, else $\langle [\![\mathsf{Does}]\!](a, \beta), t_1, t_2 \rangle \in [\![\mathsf{Occurs}]\!]$
   - For every $a \in \mathfrak{A}$ and $\langle [\![\mathsf{Does}]\!](a, [\![\mathsf{While}]\!](s, \alpha)), t_1, t_2 \rangle \in [\![\mathsf{Occurs}]\!]$, $\langle [\![\mathsf{Does}]\!](a, [\![\mathsf{If}]\!](s, [\![\mathsf{Seq}]\!](\alpha, [\![\mathsf{While}]\!](s, \alpha)), \mathsf{NoOp})), t_1, t_2 \rangle \in [\![\mathsf{Occurs}]\!]$.

For completeness, an $\mathcal{L}$-theory will include axioms capturing the above constraints on interpretation. I also assume the existence of predicate and function symbols to represent preconditions and effects of actions. However, I do not take the agent to be a planning agent; rather, the agent is provided with $\mathcal{L}$-representations of recipes of action to achieve desired states. These assumptions are, nonetheless, totally harmless, given the nature of our task.

4.3 The PML

The language $\mathcal{L}$, comprising the symbolic structures at the KL, is an objective, non-indexical, first-order language. As such, it suffices for reasoning *about* action and time. But as the examples of Section 1 attest, more is needed. To arrive at an adequate treatment of indexicality, we need to now turn to the PML. We may describe the relevant aspects of the PML using the notion of a PML state.

Definition 1 *A PML state is a quadruple* $\mathbb{P} = \langle \Pi, \gamma, \Sigma, {}^*\mathsf{NOW}\rangle$, *where*

1. Π *is a set of PML representations (typically, feature vectors) of perceivable entities (objects, properties, and relations), and behaviors that directly control the SAL.*
2. γ*, the grounding relation, is set of pairs of* $\mathcal{L}$*-terms and members of* Π. *In particular,* γ *is functional and left-total on the set of atomic* σ_A*-terms, mapping each such term* a *into a routine* $\gamma(a)$.
3. Σ *is a sequence of* σ_A*-terms representing acts scheduled for execution.*
4. NOW *is a PML variable, whose value, at any time, is a* σ_T*-constant. The* * *is a de-referencing operator and, hence,* ${}^*\mathsf{NOW}$ *is the* σ_T*-constant which is the value of* NOW *in the state.*

The σ_T-constant ${}^*\mathsf{NOW}$ denotes the current time, for the agent, in a given PML state. This term is distinguished in practical reasoning by being the value of the variable NOW.

4.4 Dynamics

Reasoning, acting, and perception change the state of the agent. Such changes are governed by a set of rules. In the case of a reasoning-only agent, these are logical rules of inference. In the case of a reasoning *and* acting agent, we need to generalize the notion of an inference rule. When interpreted operationally, classical rules of inference transform one *belief* state into another. Hence, the first step in generalizing inference rules is to generalize the notion of a state.

Definition 2 *An agent state is a triple* $\mathbb{S} = \langle \mathbb{K}, \mathbb{D}, \mathbb{P}\rangle$*, where* $\mathbb{K}$ *is a set of* $\mathcal{L}$*-WFFs,* $\mathbb{D}$ *is a set of* $\mathcal{L}$*-WFDs, and* $\mathbb{P}$ *is a PML state.*

A practical inference cannon is a mapping (which is not necessarily functional) from agent states to agent states. A common way of viewing this mapping is as a set of transformation rules on agent states. Such rules sometimes have preconditions and side effects. Rules will be displayed as follows

$$Pre|\langle \mathbb{K}, \mathbb{D}, \mathbb{P}\rangle \longrightarrow \langle \mathbb{K}', \mathbb{D}', \mathbb{P}'\rangle|Eff$$

where, Pre is a set of preconditions and Eff is a set of effects. Preconditions are typically conditions on $\mathcal{L}$-terms appearing in $\mathbb{K}$; effects are exclusively of the form $Initiate(\beta)$, where β is a PML behavior in Π. This indicates that a side-effect of applying the rule is for the agent to start carrying out β. A rule $Pre|\mathbb{S} \longrightarrow \mathbb{S}'|Eff$ is *applicable* to state $\mathbb{S}$ if its preconditions are satisfied.

Definition 3 *Let* $r = Pre|\langle \mathbb{K}, \mathbb{D}, \mathbb{P}\rangle \longrightarrow \langle \mathbb{K}', \mathbb{D}', \mathbb{P}'\rangle|Eff$ *be a rule with* $\mathbb{P} = \langle \Pi, \gamma, \Sigma, {}^*\mathsf{NOW}\rangle$ *and* $\mathbb{P}' = \langle \Pi', \gamma', \Sigma', {}^*\mathsf{NOW}'\rangle$.

1. r *is said to be an inference rule if* $\mathbb{D}' = \mathbb{D}$, $\mathbb{P}' = \mathbb{P}$, *and* $Eff = \varnothing$. $\mathbb{K}$ *is deductively-closed, denoted* $\mathbb{K} = \mathrm{Cn}(\mathbb{K})$ *if it is closed under the application of all inference rules.*
2. r *is said to be a decomposition rule if* $\mathbb{K}' = \mathbb{K} = \mathrm{Cn}(\mathbb{K})$, $\mathbb{D}' = \mathbb{D}$, $\Pi = \Pi'$, $\gamma = \gamma'$, ${}^*\mathsf{NOW} = {}^*\mathsf{NOW}'$, $\Sigma \neq \langle\rangle$ *and* $Eff = \varnothing$.
3. r *is an initiation rule if* $\mathbb{K} = \mathrm{Cn}(\mathbb{K})$, $\mathbb{D}' = \mathbb{D}$, $\Pi = \Pi'$, $\gamma = \gamma'$, $\Sigma \neq \langle\rangle$, *and* $Eff \neq \varnothing$.
4. r *is a directive rule if* $\mathbb{K}' = \mathbb{K} = \mathrm{Cn}(\mathbb{K})$, $\Pi = \Pi'$, $\gamma = \gamma'$, ${}^*\mathsf{NOW} = {}^*\mathsf{NOW}'$, $\Sigma = \langle\rangle$, *and* $\mathbb{D} \neq \varnothing$

As is customary, the above rules define a *yielding* relation between agent states.

Definition 4 *An agent state* $\mathbb{S}$ *yields an agent state* $\mathbb{S}$*', denoted* $\mathbb{S} \longrightarrow^+ \mathbb{S}'$*, if there is sequence of states* $\mathbb{S}_1, \ldots, \mathbb{S}_n$*, such that* $\mathbb{S}_1 = \mathbb{S}$, $\mathbb{S}_n = \mathbb{S}'$*, and, for every* $1 \leq i < n$*, there is a rule* $Pre|\mathbb{S}_i \longrightarrow \mathbb{S}_{i+1}|Eff$ *which is applicable to* $\mathbb{S}_i$*. A sequence of rules taking the agent from state* $\mathbb{S}$ *to state* $\mathbb{S}'$ *via the intermediate states is called an* $(\mathrm{S}, \mathrm{S}')$*-path.*

Definition 3 imposes a strict order on the application of rules: all applicable inference rules must be first applied, followed by decomposition and initiation rules, and finally followed by directive rules.

Observation 4.1 *Let* $\mathbb{S}$ *be an agent state.*

1. *If there is a state* $\mathbb{S}'$ *and an* $(\mathrm{S}, \mathrm{S}')$*-path which is a sequence of inference rules, then no decomposition, initiation, or directive rule is applicable to* $\mathbb{S}$.
2. *If there is a state* $\mathbb{S}'$ *and an* $(\mathrm{S}, \mathrm{S}')$*-path which is a sequence of inference, decomposition, and initiation rules, then no directive rule is applicable to* $\mathbb{S}$.

Figure 1 shows the set of rules we consider for our agent. Absent from this figure, and the entire discussion, is any mention of perception. Perception results in changing the agent state when PML routines read-off the values of SAL sensors and interpret them by constructing PML representations. The effect of perception is primarily on Π, γ, $\mathbb{K}$, ${}^*\mathsf{NOW}$, and possibly $\mathbb{D}$ (see [26, 10]).

The rules in Figure 1 embody (at least) three simplifying (but totally inert) assumptions about our agent:

1. Atomic actions are punctual and immediately successful. A more careful approach is, in general, called for. (See [8, 9].)
2. Time moves (i.e., ${}^*\mathsf{NOW}$ changes) only when the agent acts. We could have chosen otherwise. For example, following [6], each rule may be defined to change ${}^*\mathsf{NOW}$ akin to the initiation rule. Alternatively, we may have rules dedicated to changing ${}^*\mathsf{NOW}$, which are applied synchronously with a PML *pacemaker* [26]. Whatever the choice, not much depends on it when it comes to the proposed treatment of indexicality.
3. A solution to the frame problem needs to be incorporated in order to account for which states persist and which do not as a result of applying an initiation rule. Again, this is not the main concern here, and a monotonic solution to the frame problem (e.g., [24]) will suffice for our purposes.

Given these rules, we can prove that, the agent's beliefs can only expand as time unfolds and that, as far as the agent is concerned, time unfolds from the past to the future.

Proposition 4.1 *If* $\mathbb{S} \longrightarrow^+ \mathbb{S}'$ *then*

1. $\mathbb{K} \subseteq \mathbb{K}'$ *and*
2. ${}^*\mathsf{NOW} = {}^*\mathsf{NOW}'$ *or* ${}^*\mathsf{NOW} \prec {}^*\mathsf{NOW}' \in \mathbb{K}'$.

Since we assume a sound and complete set of inference rules, the agent's reasoning will be sound and complete given a correct and complete axiomatization of the domain-independent symbols of $\mathcal{L}$. However, we also need to verify that the rules guide the agent to correct execution of actions. To this end, we need some terminology.

Inference Rules. Any set of monotonic, first-order rules which is sound and complete for $\mathcal{L}$.

Decomposition Rules. In what follows, $\odot$ represents sequence concatenation.

1. $\{\mathbb{K} = \mathrm{Cn}(\mathbb{K})\}|\langle \mathbb{K}, \mathbb{D}, \langle \Pi, \gamma, \mathsf{NoOp} \odot \Sigma, {}^*\mathsf{NOW}\rangle\rangle \longrightarrow \langle \mathbb{K} \cup \{\mathsf{Occurs}(\mathsf{Does}(\mathsf{I}, \mathsf{NoOp}), {}^*\mathsf{NOW}, {}^*\mathsf{NOW})\}, \mathbb{D}, \langle \Pi, \gamma, \Sigma, {}^*\mathsf{NOW}\rangle\rangle|\varnothing$
2. $\{\mathbb{K} = \mathrm{Cn}(\mathbb{K})\}|\langle \mathbb{K}, \mathbb{D}, \langle \Pi, \gamma, \mathsf{Seq}(\alpha, \beta) \odot \Sigma, {}^*\mathsf{NOW}\rangle\rangle \longrightarrow \langle \mathbb{K}, \mathbb{D}, \langle \Pi, \gamma, \alpha \odot \beta \odot \Sigma, {}^*\mathsf{NOW}\rangle\rangle|\varnothing$
3. $\{\mathbb{K} = \mathrm{Cn}(\mathbb{K}), \mathsf{Holds}(s, {}^*\mathsf{NOW}) \in \mathbb{K}\}| \langle \mathbb{K}, \mathbb{D}, \langle \Pi, \gamma, \mathsf{If}(s, \alpha, \beta) \odot \Sigma, {}^*\mathsf{NOW}\rangle\rangle \longrightarrow \langle \mathbb{K}, \mathbb{D}, \langle \Pi, \gamma, \alpha \odot \Sigma, {}^*\mathsf{NOW}\rangle\rangle|\varnothing$
4. $\{\mathbb{K} = \mathrm{Cn}(\mathbb{K}), \mathsf{Holds}(s, {}^*\mathsf{NOW}) \notin \mathbb{K}\}| \langle \mathbb{K}, \mathbb{D}, \langle \Pi, \gamma, \mathsf{If}(s, \alpha, \beta) \odot \Sigma, {}^*\mathsf{NOW}\rangle\rangle \longrightarrow \langle \mathbb{K}, \mathbb{D}, \langle \Pi, \gamma, \beta \odot \Sigma, {}^*\mathsf{NOW}\rangle\rangle|\varnothing$
5. $\{\mathbb{K} = \mathrm{Cn}(\mathbb{K}), \mathsf{Holds}(s, {}^*\mathsf{NOW}) \in \mathbb{K}\}| \langle \mathbb{K}, \mathbb{D}, \langle \Pi, \gamma, \mathsf{While}(s, \alpha) \odot \Sigma, {}^*\mathsf{NOW}\rangle\rangle \longrightarrow \langle \mathbb{K}, \mathbb{D}, \langle \Pi, \gamma, \alpha \odot \mathsf{While}(s, \alpha) \odot \Sigma, {}^*\mathsf{NOW}\rangle\rangle|\varnothing$
6. $\{\mathbb{K} = \mathrm{Cn}(\mathbb{K}), \mathsf{Holds}(s, {}^*\mathsf{NOW}) \notin \mathbb{K}\}| \langle \mathbb{K}, \mathbb{D}, \langle \Pi, \gamma, \mathsf{While}(s, \alpha) \odot \Sigma, {}^*\mathsf{NOW}\rangle\rangle \longrightarrow \langle \mathbb{K}, \mathbb{D}, \langle \Pi, \gamma, \Sigma, {}^*\mathsf{NOW}\rangle\rangle|\varnothing$

Initiation Rule.

$$\{\alpha \text{ is atomic}, \mathbb{K} = \mathrm{Cn}(\mathbb{K}), t_2 \text{ and } t_3 \text{ appear nowhere in } \mathbb{K}\}| \langle \mathbb{K}, \mathbb{D}, \langle \Pi, \gamma, \alpha \odot \Sigma, t_1\rangle\rangle \longrightarrow \langle \mathbb{K} \cup \{t_1 \prec t_2, t_2 \prec t_3, \mathsf{Occurs}(\mathsf{Does}(\mathsf{I}, \alpha), t_2, t_2)\}, \mathbb{D}, \langle \Pi, \gamma, \Sigma, t_3\rangle\rangle|\{Initiate(\gamma(\alpha))\}$$

Directive Rules.

1. $\{\mathbb{K} = \mathrm{Cn}(\mathbb{K}), \mathsf{Holds}(s, {}^*\mathsf{NOW}) \in \mathbb{K}\}| \langle \mathbb{K}, \{\mathsf{Whenever}(s, \alpha)\} \cup \mathbb{D}, \langle \Pi, \gamma, \langle\rangle, {}^*\mathsf{NOW}\rangle\rangle \longrightarrow \langle \mathbb{K}, \{\mathsf{Whenever}(s, \alpha)\} \cup \mathbb{D}, \langle \Pi, \gamma, \langle\alpha\rangle, {}^*\mathsf{NOW}\rangle\rangle|\varnothing$
2. $\{\mathbb{K} = \mathrm{Cn}(\mathbb{K}), \mathsf{Holds}(s, {}^*\mathsf{NOW}) \in \mathbb{K}\}| \langle \mathbb{K}, \{\mathsf{When}(s, \alpha)\} \cup \mathbb{D}, \langle \Pi, \gamma, \langle\rangle, {}^*\mathsf{NOW}\rangle\rangle \longrightarrow \langle \mathbb{K}, \mathbb{D}, \langle \Pi, \gamma, \langle\alpha\rangle, {}^*\mathsf{NOW}\rangle\rangle|\varnothing$

Figure 1. Rules of Practical Inference

Definition 5 *Let $\mathbb{S}$ and $\mathbb{S}'$ be agent states and p be an $(\mathbb{S}, \mathbb{S}')$-path. If $\langle r_1, \ldots, r_n\rangle$ is the (longest) subsequence of p of instances of the initiation rule, then the action trace of p, denoted $\mathfrak{tr}(p)$, is the sequence $\langle (t_1, \alpha_1), \ldots, (t_n, \alpha_n)\rangle$, where, for $1 \leq i \leq n$,*

$$r_i = Pre|\mathbb{S}_i \longrightarrow \langle \mathbb{K} \cup \{t_{i1} \prec t_i, t_i \prec t_{i3}, \mathsf{Occurs}(\mathsf{Does}(\mathsf{I}, \alpha_i), t_i, t_i)\}, \mathbb{D}, \langle \Pi, \gamma, \Sigma, t_{i3}\rangle\rangle|\{Initiate(\gamma(\alpha_i))\}$$

for some agent state $\mathbb{S}_i$.

The following result immediately follows.

Proposition 4.2 *If p is an $(\mathbb{S}, \mathbb{S}')$-path, then $\mathfrak{tr}(p)$ is unique.*

Proof. This follows since $\mathbb{K}'$ carries the history of which atomic actions were performed when, given the monotonicity of inference and the WFF $\mathsf{Occurs}(\mathsf{Does}(\mathsf{I}, \alpha_i), t_i, t_i)\}$ added by the initiation rule. □

Definition 6 *Let $\mathfrak{M}$ be an $\mathcal{L}$-structure and let p be an $(\mathbb{S}, \mathbb{S}')$-path. $\mathfrak{M}$ is p-faithful if, for every (t_i, α_i) in $\mathfrak{tr}(p)$, $\mathfrak{M} \models \mathsf{Occurs}(\mathsf{Does}(\mathsf{I}, \alpha_i), t_i, t_i)$.*

For every, $(\mathbb{S}, \mathbb{S}')$-path, there is a special class of faithful structures.

Observation 4.2 *Let p be an $(\mathbb{S}, \mathbb{S}')$-path, where $\mathbb{S}' = \langle \mathbb{K}', \mathbb{D}', \mathbb{P}'\rangle$. If $\mathfrak{M}$ is an $\mathcal{L}$-structure such that $\mathfrak{M} \models \mathbb{K}'$, then $\mathfrak{M}$ is p-faithful.*

Proof. This follows from Proposition 4.2 and the first clause of Proposition 4.1. □

Hence, we can now prove that our rules, not only guide our agent to sound reasoning, but also to correct action.

Theorem 1 *Let $\mathbb{S} = \langle \mathbb{K}, \{\mathsf{Whenever}(s, \alpha)\} \cup \mathbb{D}, \mathbb{P}\rangle$ ($\mathbb{S} = \langle \mathbb{K}, \{\mathsf{When}(s, \alpha)\} \cup \mathbb{D}, \mathbb{P}\rangle$) be an agent state to which the first (respectively, second) directive rule is applicable. Then there is a state $\mathbb{S}'$ such that $\mathbb{S} \longrightarrow^+ \mathbb{S}'$ and, for every structure $\mathfrak{M}$, if $\mathfrak{M} \models \mathbb{K}'$, then $\mathfrak{M} \models \mathsf{Occurs}(\mathsf{Does}(\mathsf{I}, \alpha), {}^*\mathsf{NOW}, {}^*\mathsf{NOW}')$.*

Proof. The proof starts by noting that $\mathfrak{M}$ is a model of $\mathbb{K}$ (Proposition 4.1) and is faithful to any $(\mathbb{S}, \mathbb{S}')$-path (Observation 4.2). We proceed by induction on the structure of α, given the rules of Figure 1 and the semantics of composite actions (Section 4.2). □

5 EVALUATION

To evaluate the proposed treatment of "now", we rate it against the four criteria of Section 3.2.

N1. An account of post-it-note "now" is embodied in the directive rules of Figure 1. Tense-marker "now" is accounted for since the present is always distinguished as being denoted by ${}^*\mathsf{NOW}$. The rules of Figure 1 provide the link between tense-marker "now" and action.

N2. Temporal progression is accounted for by the initiation rule.

N3. No special heavy computation is mandated by the account of "now"; ${}^*\mathsf{NOW}$ changes seamlessly at the PML, leaving the knowledge base intact as time goes by.

N4. No special assumptions about the ontology nor about the agent are made by the proposed treatment. In particular, unlike active logic [6], no assumptions about the structure of time are made; and, unlike Lespérance and Levesque's treatment, no awkward account of equality nor strange belief updates are required.

To get a feel of the system in action, we consider the case of the messy shopper from Section 1. Consider an agent state $\mathbb{S}$ satisfying the following.

- $\mathsf{Name}(\mathsf{I}, Perry) \in \mathbb{K} = \mathrm{Cn}(\mathbb{K})$.
- $\mathsf{Holds}(\mathsf{Clk}(12:00), t_1) \in \mathbb{K}$.
- $\mathsf{Holds}(Messy(\mathsf{I}), t_1) \notin \mathbb{K}$.
- $\mathsf{Whenever}(Messy(\mathsf{I}), FixIt) \in \mathbb{D}$.
- $\Sigma = \langle\rangle$.
- ${}^*\mathsf{NOW} = t_1$.

In this state, the first directive rule is not applicable since the agent does not believe that he is *now* messy. Now, consider another state $\mathbb{S}'$ which is identical to $\mathbb{S}$ except that

- $\mathbb{K}' = \mathbb{K} \cup \{\mathsf{Holds}(Messy(c1), t_2), Author(c_1, PEI), t_1 \prec t_2, \mathsf{Holds}(\mathsf{Clk}(12:01), t_2)\}$.

- $^*\mathsf{NOW}' = t_2$.

Here $Author(c_1, PEI)$ indicates that $[\![c_1]\!]$ is the author of 'the problem of the essential indexical". Again, the directive rule is not applicable since the agent does not believe that *he* is now messy. This may be fixed if the agent comes to believe that *he* is the indicated author. Assuming that the messiness of the author persists, the agent reaches a state $\mathbb{S}''$ which is identical to $\mathbb{S}'$ except that

- $\mathbb{K}'' = \mathbb{K}' \cup \{\mathsf{I} = c_1, \mathsf{Holds}(Messy(c1), t_3), \mathsf{Holds}(Messy(I), t_3), t_2 \prec t_3, \mathsf{Holds}(\mathsf{Clk}(12:02), t_3)\}$.
- $^*\mathsf{NOW}'' = t_3$

In this state, the first directive is applicable, resulting in the agent's fixing the mess he is causing. Note that in all cases, knowledge of clock time is totally irrelevant to action. It would have been, however, had the agent adopted the directive that it should fix the mess anyway once it becomes 12:05, for instance.

6 CONCLUSION

Though indexicality is indeed essential for rational action, a language of thought with indexical expressions is not. The indexical effect may be achieved through rules of practical reasoning. I have outlined a treatment of indexicality within the framework of a grounded layered agent architecture. The top layer comprises a classical non-indexical language of thought. Indexicality features in the interaction between the top layer and a lower perceptuo-motor layer which grounds action terms and the feel for temporal progression. The proposed treatment appears to be adequate at least as far as it seamlessly provides a motivated account for temporal progression and for the functions of "now" as a tense marker and a placeholder for future times in mental notes of action.

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