

# Lecture 14

CSE 331

Sep 27, 2019

If you need it, ask for help



# Mini Project group due Monday!

## CSE 331 Mini project choices

Fall 2019

Please check the table below before submitting your mini project team composition to make sure your case study is not being used by another group. Case studies are assigned on a first come first serve basis.

Group	Chosen Algorithm	Case Study	Links
Daniel Shekhtman, William Nicholson, Andrew Quinonez (D's Get Degrees)	PageRank	Manipulation of PageRank for nefarious purposes	<a href="#">Link 1</a> , <a href="#">Link 2</a> , <a href="#">Link 3</a> , <a href="#">Link 4</a>
Jordan Clemons, Chris Burton, Christopher Perez (Group 1)	Pagerank	Google's use of Pagerank in sorting search results	<a href="#">Link 1</a> , <a href="#">Link 2</a>
Moulid Ahmed, Shrishty Shivani Jha, Shreya Lakhkar (ACE-MA)	Spotify Recommendation	Machine Learning Algorithm	<a href="#">Link 1</a> , <a href="#">Link 2</a> , <a href="#">Link 3</a>
Justin Henderson, Hannah Wlasowicz, Judy Mei (PizzaTime)	Aes 256	ransomware	<a href="#">Link 1</a>
Gillian Marcus, Jason Niu, Sharon Stack (2n^2 (//pls substitute caret for a superscript))	Deep Neural Networks for YT Recommendations	Social Media Targeted Advertising	<a href="#">Link 1</a> , <a href="#">Link 2</a> , <a href="#">Link 3</a> , <a href="#">Link 4</a>

# The chosen list updates ~2 days

note ☆ stop following 23 views

## Please only submit your video choice ONCE

The Google form does not automatically update the algo choices: <http://www-student.cse.buffalo.edu/~atri/cse331/fall19/mini-project/algos.html>

I update it every two days. If your choice is not updated above please do NOT submit again-- it just means more work for me since in this case I have to manually update the form sheet. If you do not see an update in 2 days, please post on piazza to check: thanks!

#pin

mini\_project

edit · good note | 0

Updated 1 minute ago by Atri Rudra

# HW 4 out

## Homework 4

Due by **11:00am, Friday, October 4, 2019**.

Make sure you follow all the [homework policies](#).

All submissions should be done via [Autolab](#).

## Sample Problem

### The Problem

Extend the topological ordering algorithm [topological ordering care package](#) so that, given an input directed graph  $G$ , it outputs one of two things: (a) a topological ordering, thus establishing that  $G$  is a DAG, or (b) a cycle in  $G$ , thus establishing that  $G$  is not a DAG.

The running time of your algorithm should be  $O(m + n)$  for a directed graph with  $n$  nodes and  $m$  edges.

[Click here for the Solution](#)

## Submission

You will **NOT** submit this question. This is for you to get into thinking more about designing algorithms on graphs.

# HW 3 Solutions

At the end of the lecture

# Graded HW 2

*Hopefully* by tonight

# Questions?



# Breadth First Search (BFS)

Build layers of vertices connected to  $s$

$$L_0 = \{s\}$$

Assume  $L_0, \dots, L_j$  have been constructed

$L_{j+1}$  set of vertices not chosen yet but are connected to  $L_j$

Stop when new layer is empty

Use linked lists

Use  $CC[v]$  array

# Rest of Today's agenda

Quick run time analysis for BFS

Quick run time analysis for DFS (and Queue version of BFS)

Helping you schedule your activities for the day

# $O(m+n)$ BFS Implementation

BFS(s)

Array

Input graph as  
Adjacency list

$CC[s] = T$  and  $CC[w] = F$  for every  $w \neq s$

Set  $i = 0$

Set  $L_0 = \{s\}$

While  $L_i$  is not empty

$L_{i+1} = \emptyset$

For every  $u$  in  $L_i$

For every edge  $(u, w)$

If  $CC[w] = F$  then

$CC[w] = T$

Add  $w$  to  $L_{i+1}$

$i++$

Linked List

Version in KT  
also  
computes a  
BFS tree

# All the layers as one

BFS( $s$ )

$CC[s] = T$  and  $CC[w] = F$  for every  $w \neq s$

Set  $i = 0$

Set  $L_0 = \{s\}$

While  $L_i$  is not empty

$L_{i+1} = \emptyset$

For every  $u$  in  $L_i$

For every edge  $(u, w)$

If  $CC[w] = F$  then

$CC[w] = T$

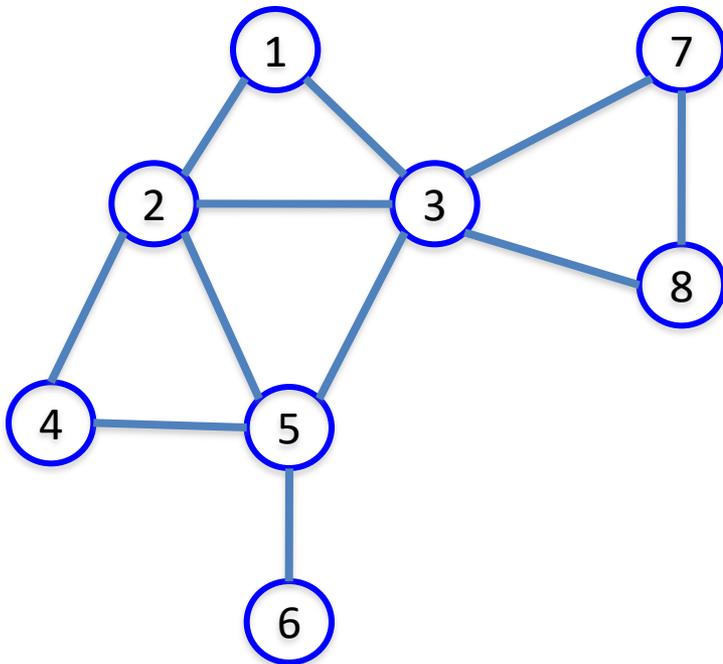
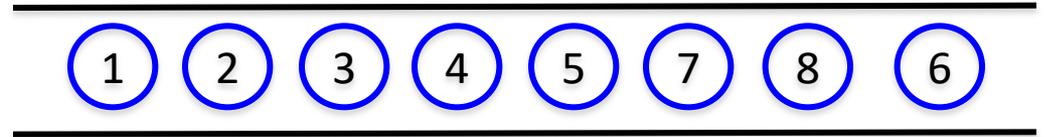
Add  $w$  to  $L_{i+1}$

$i++$

All layers are considered in first-in-first-out order

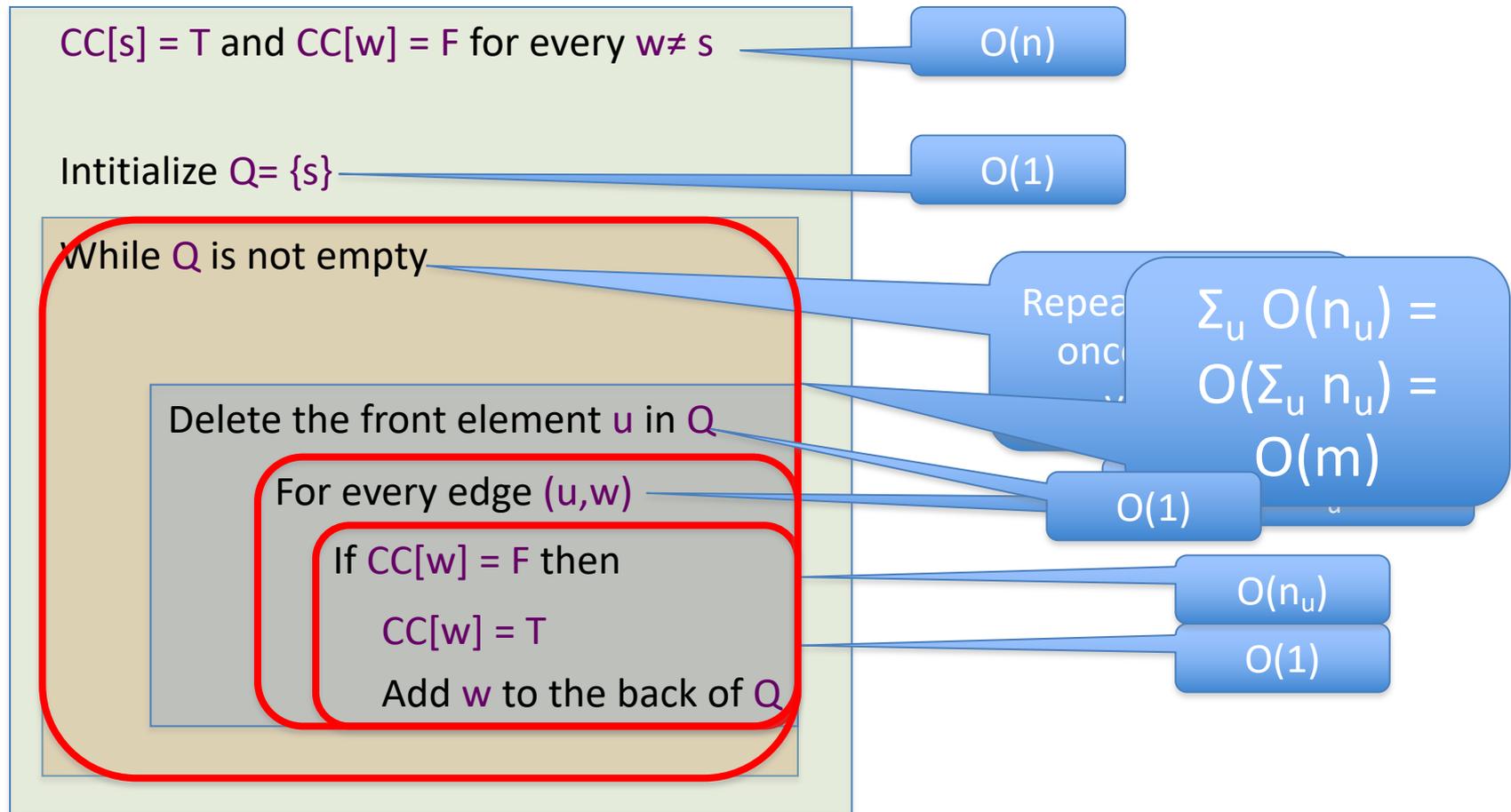
Can combine all layers into one queue: all the children of a node are added to the end of the queue

# An illustration



# Queue $O(m+n)$ implementation

BFS(s)



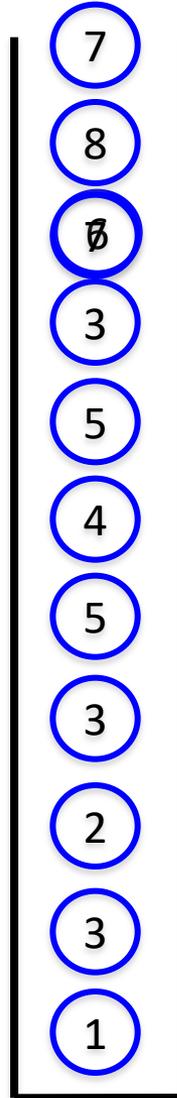
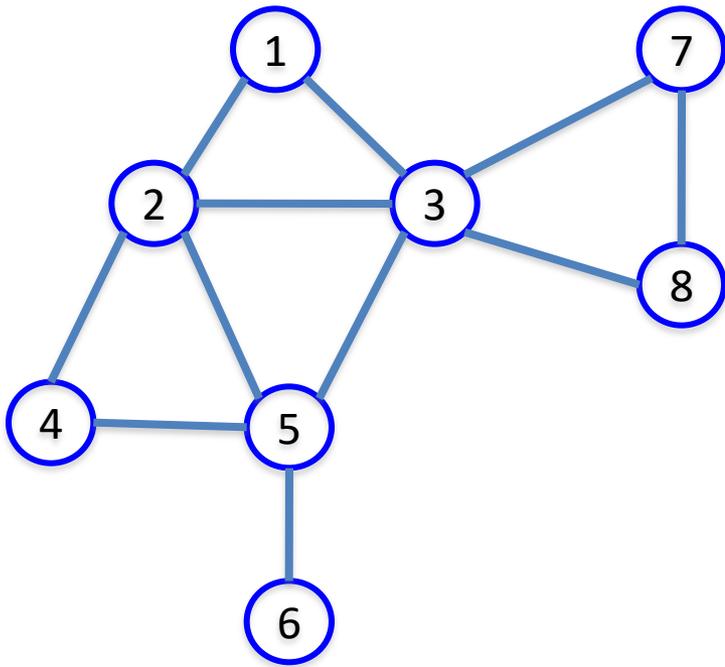
# Questions?



# Implementing DFS in $O(m+n)$ time

Same as BFS except stack instead of a queue

# A DFS run using an explicit stack



# DFS stack implementation

DFS( $s$ )

$CC[s] = T$  and  $CC[w] = F$  for every  $w \neq s$

Initialize  $\hat{S} = \{s\}$

While  $\hat{S}$  is not empty

Pop the top element  $u$  in  $\hat{S}$

For every edge  $(u,w)$

If  $CC[w] = F$  then

$CC[w] = T$

Push  $w$  to the top of  $\hat{S}$



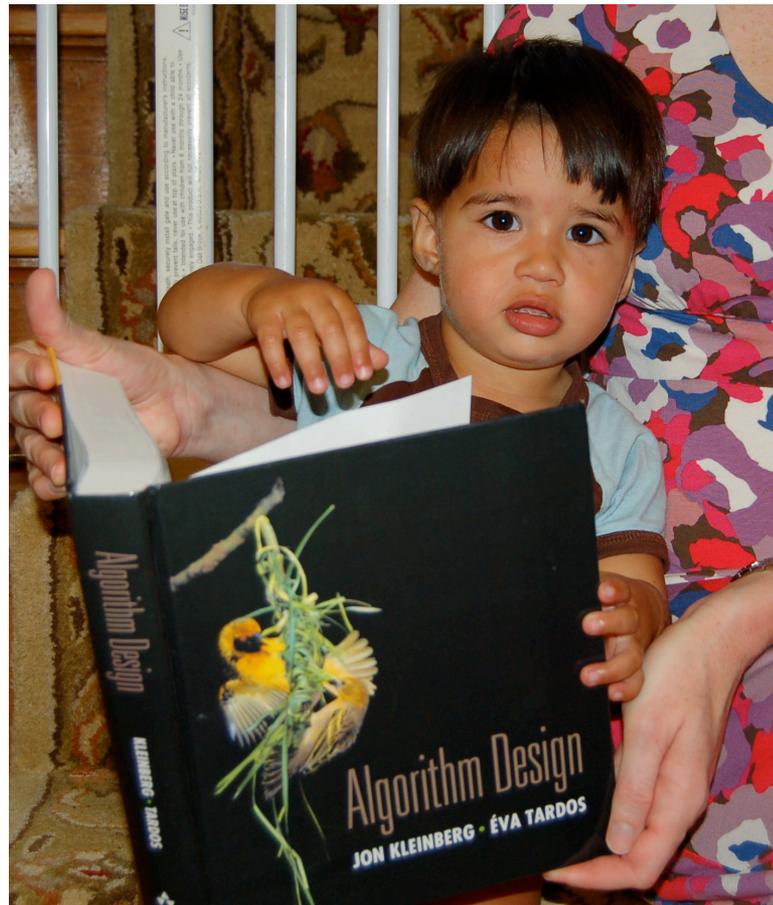
Same  
 $O(m+n)$  run  
time analysis  
as for BFS

# Questions?



# Reading Assignment

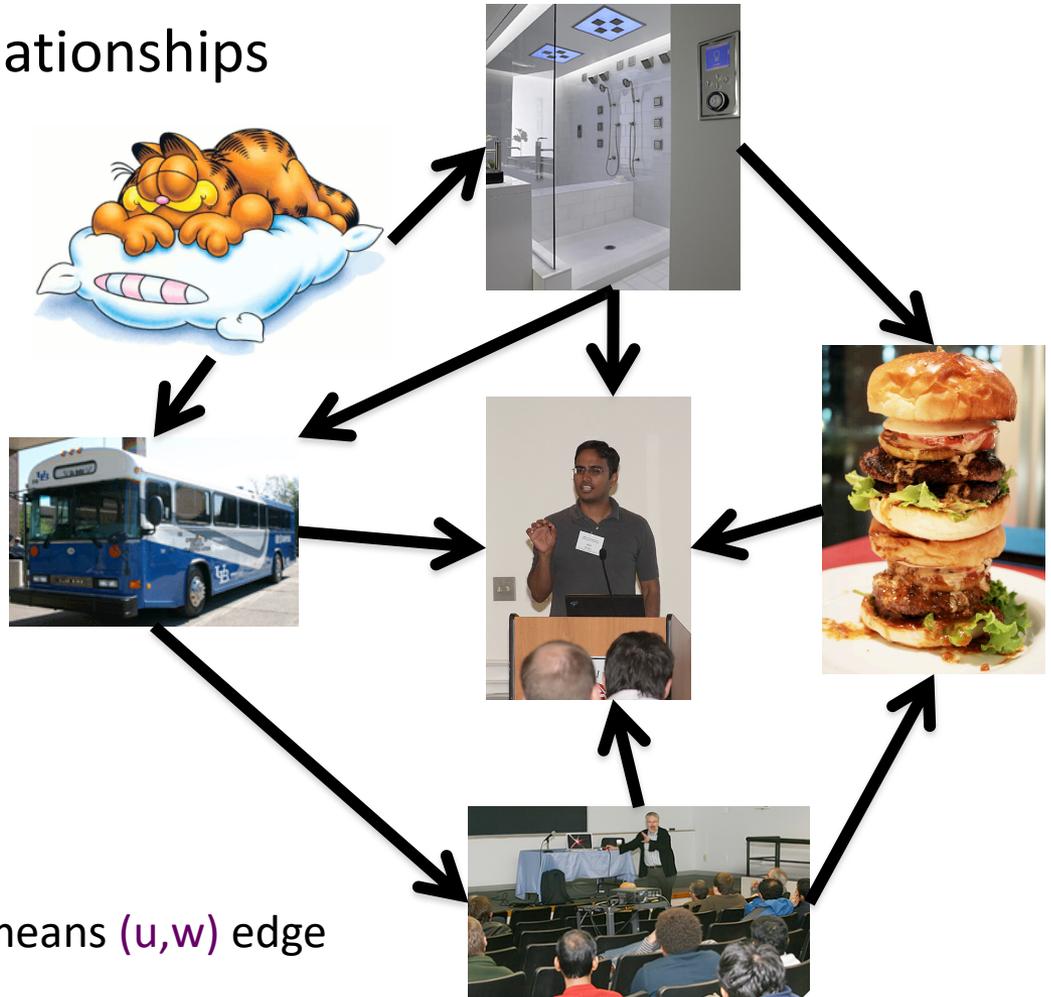
Sec 3.3, 3.4, 3.5 and 3.6 of [KT]



# Directed graphs

Model asymmetric relationships

Precedence relationships

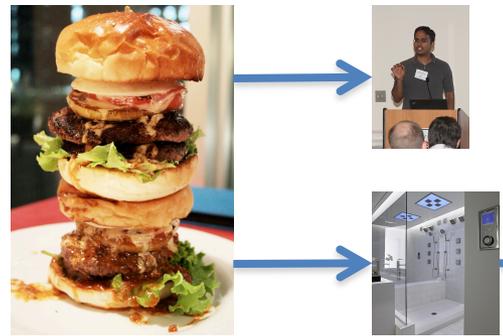
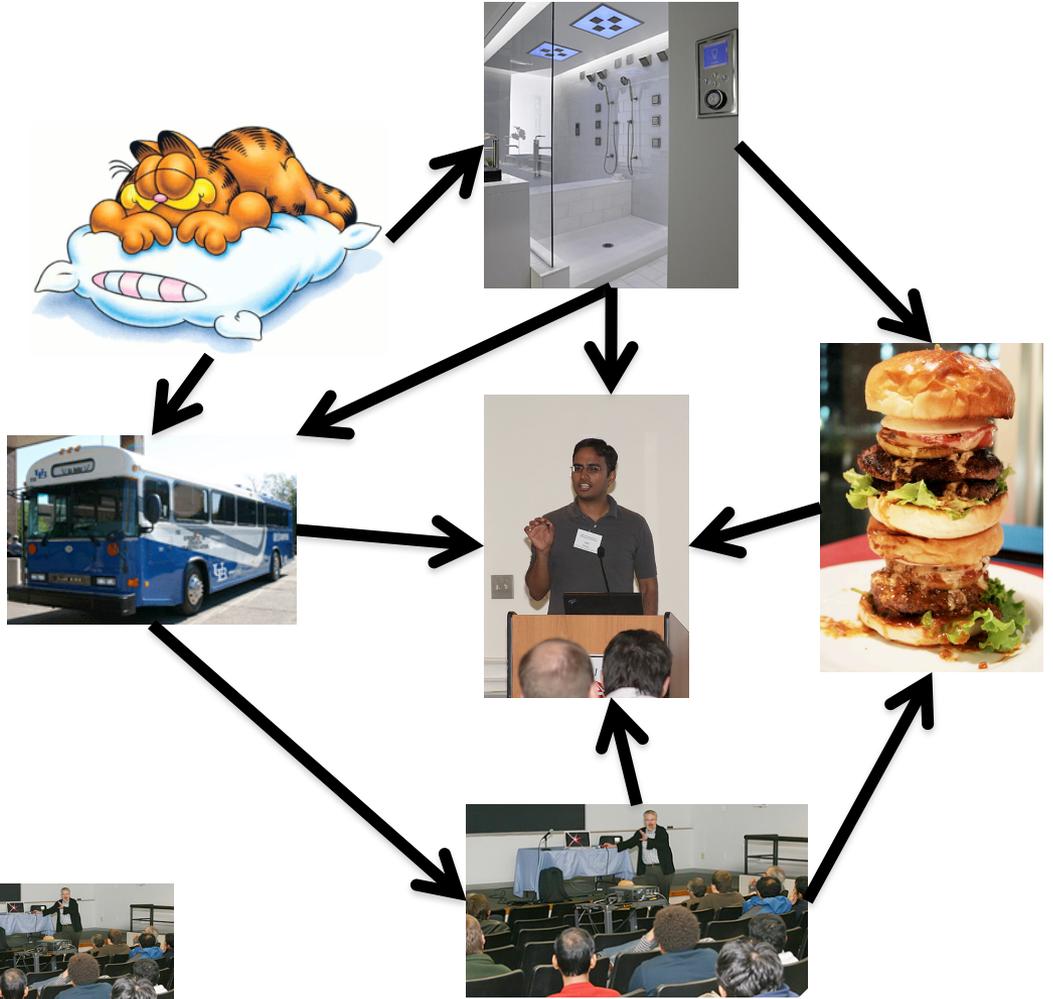


$u$  needs to be done before  $w$  means  $(u,w)$  edge

# Directed graphs

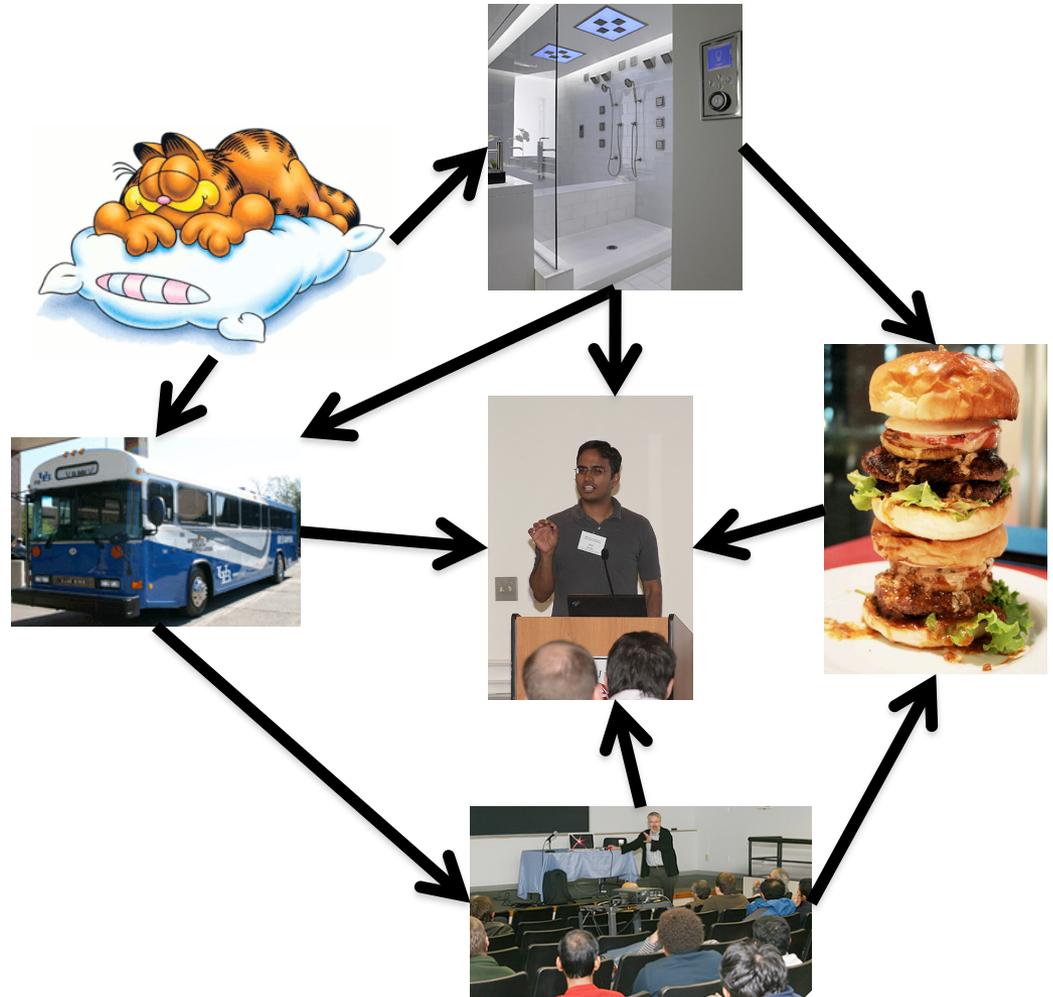
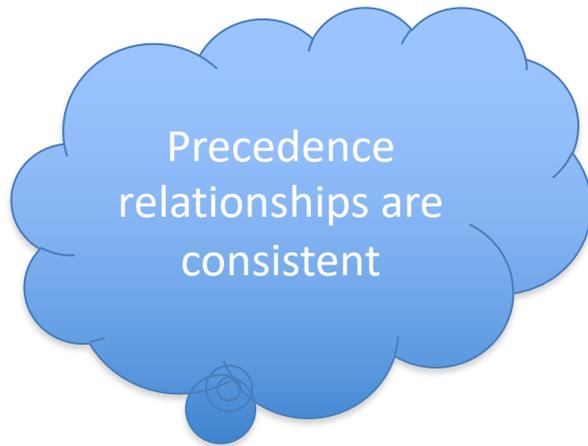
Adjacency matrix is not symmetric

Each vertex has two lists in Adj. list rep.



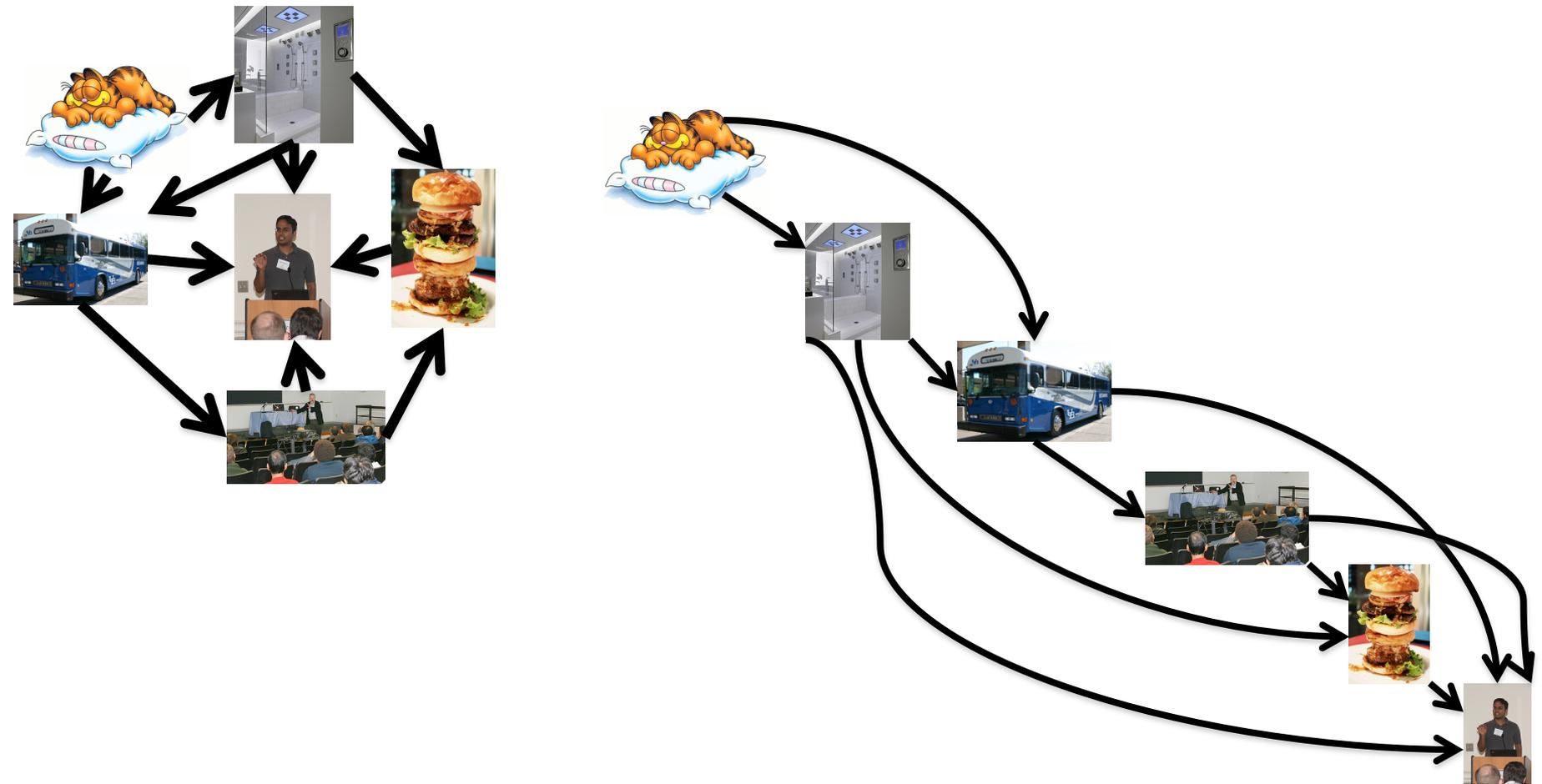
# Directed Acyclic Graph (DAG)

No directed cycles



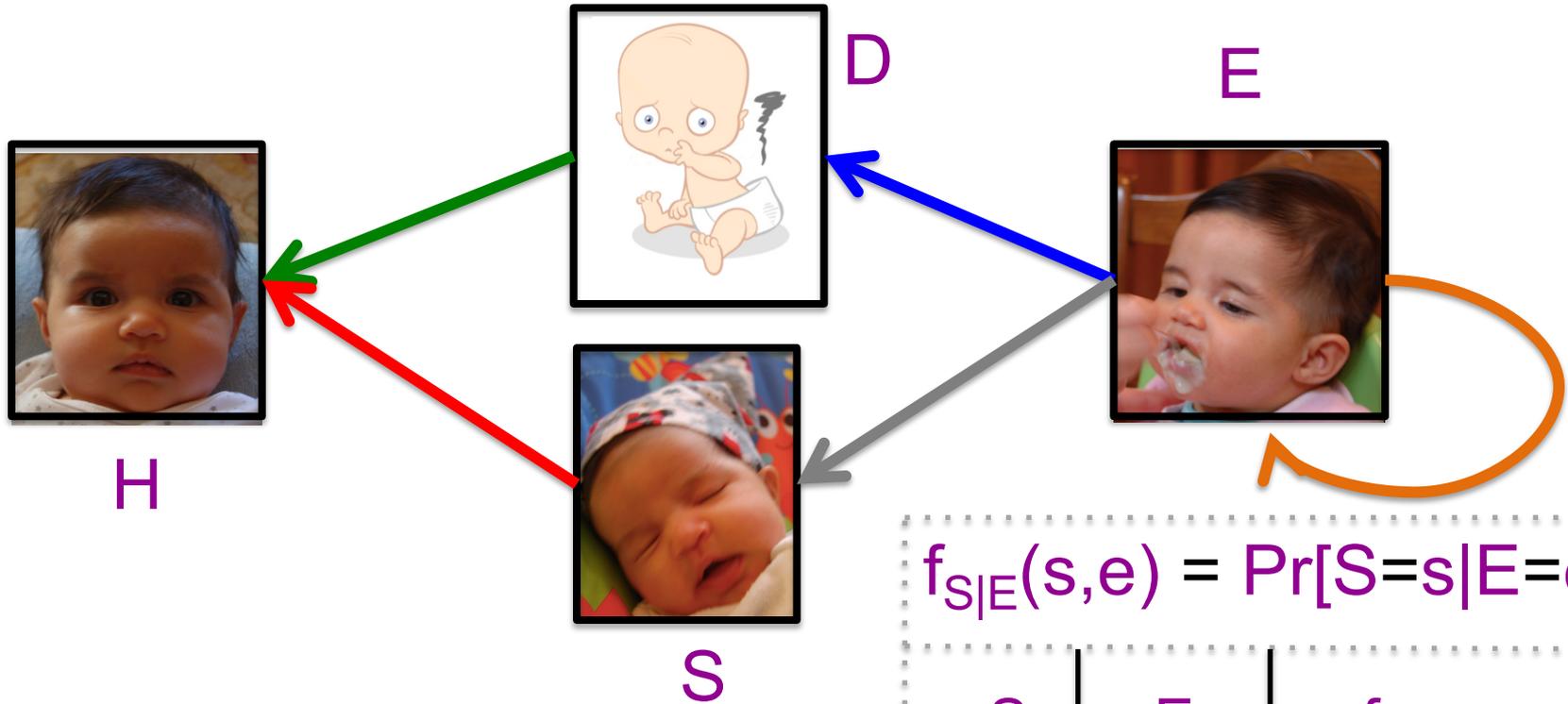
# Topological Sorting of a DAG

Order the vertices so that all edges go “forward”



# Probabilistic Graphical Models (PGMs)

<http://ginaskokopelli.com/wp-content/uploads/2013/01/DiaperDealsLogo.jpg>



$$f_{S|E}(s,e) = \Pr[S=s|E=e]$$

S	E	$f_{S E}$
1	1	0.8
1	0	0.3
0	1	0.2
0	0	0.7

$$\varphi(h) = \sum_{d,s,e} f_{H|D,S}(h,d,s) \times f_{S|E}(s,e) \times f_{D|E}(d,e) \times f_E(e)$$

# More details on Topological sort

## Topological Ordering

This page collects material from previous incarnations of CSE 331 on topological ordering.

### Where does the textbook talk about this?

**Section 3.6** in the textbook has the lowdown on topological ordering.

### Fall 2018 material

#### First lecture

Here is the lecture video:

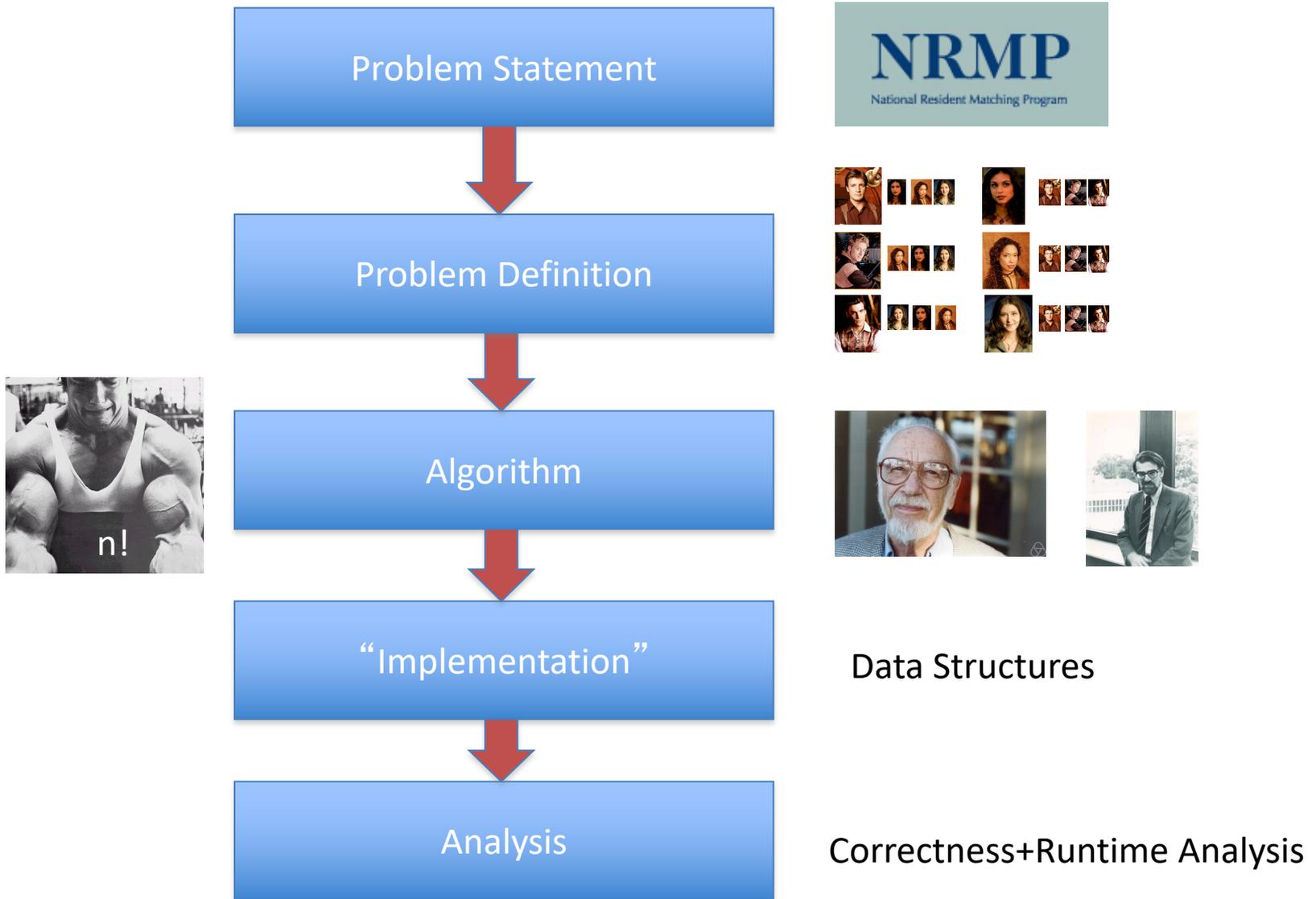
CSE331 on 10/1/2018 (Mon)



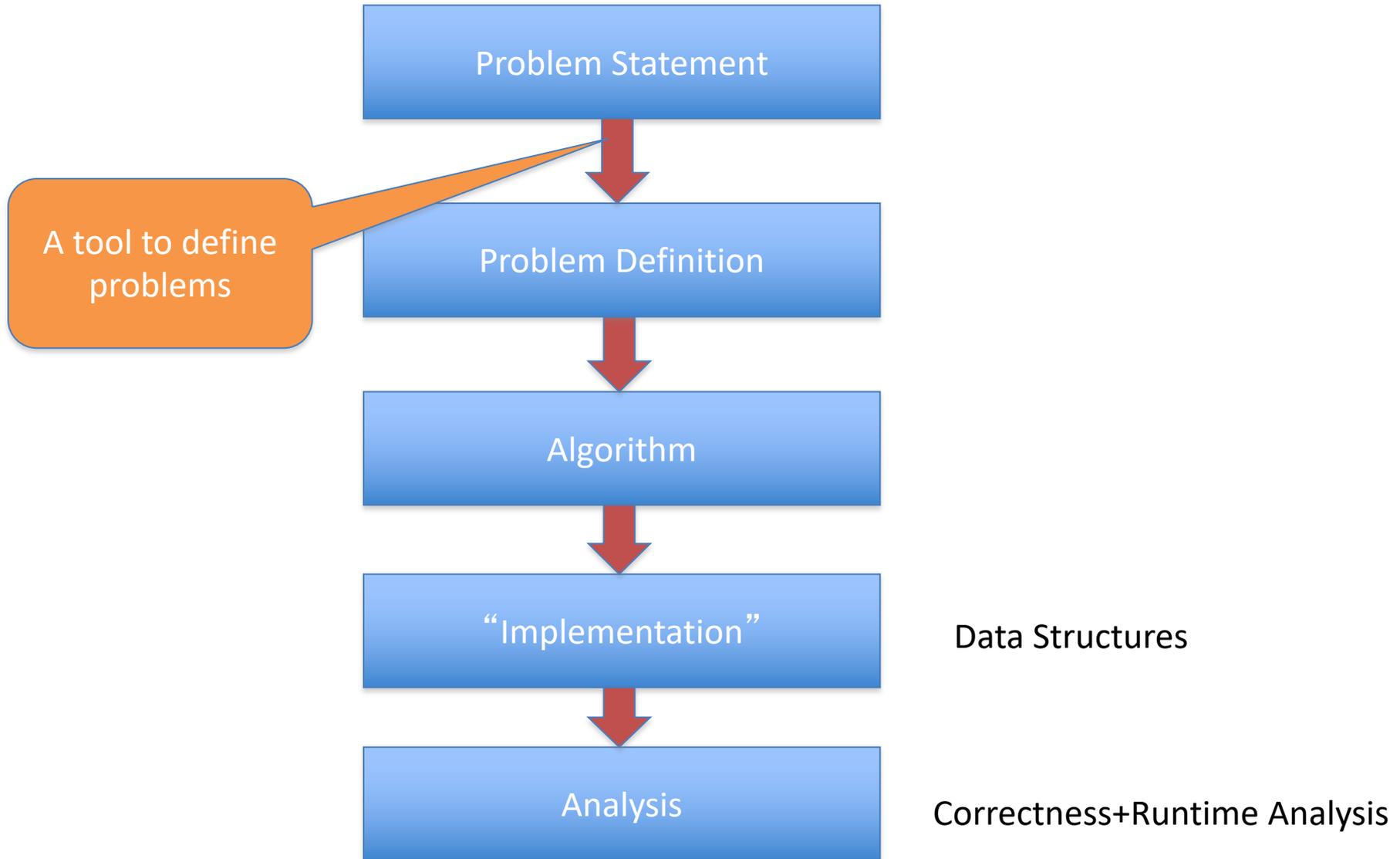
# Questions?



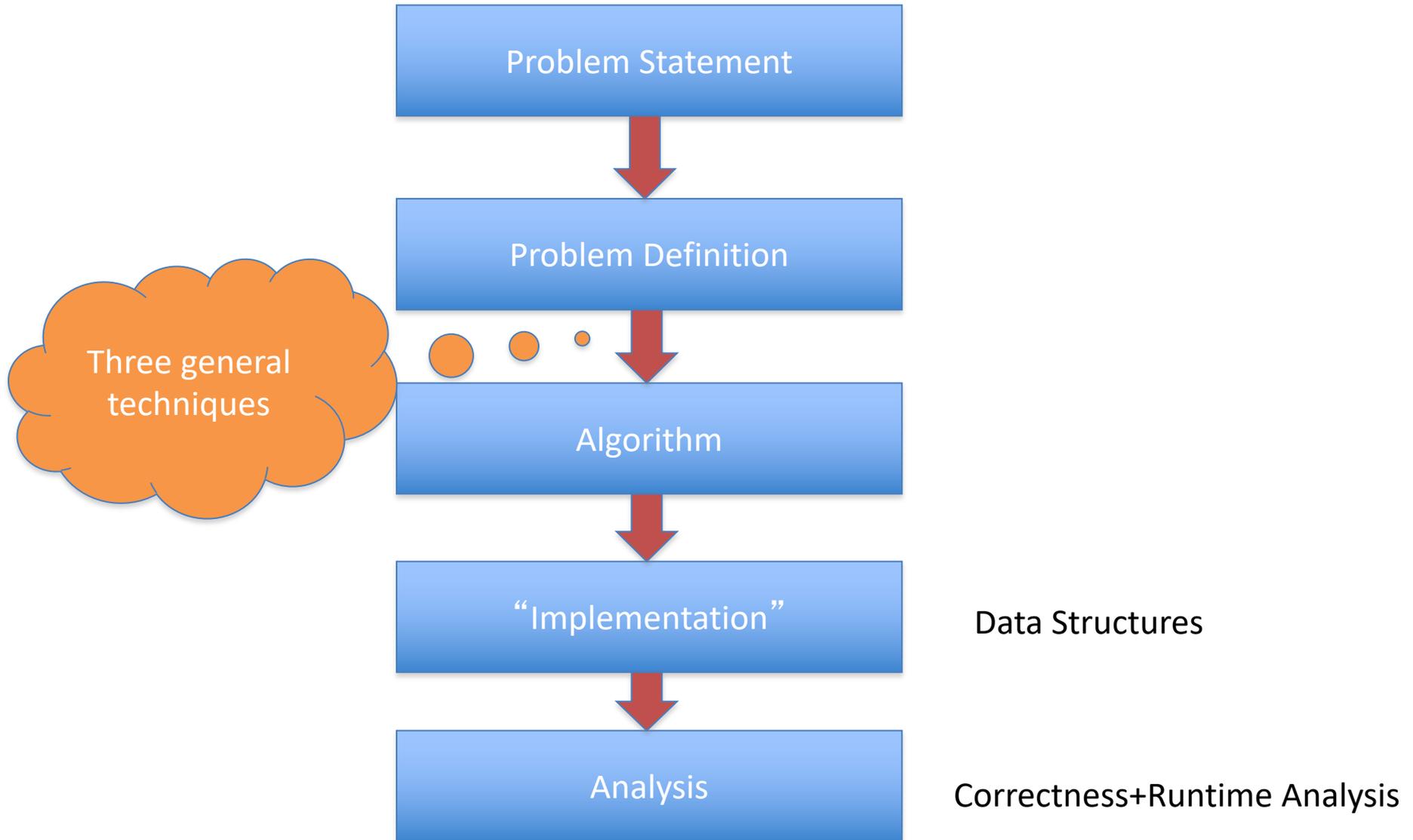
# Main Steps in Algorithm Design



# Where do graphs fit in?



# Rest of the course\*



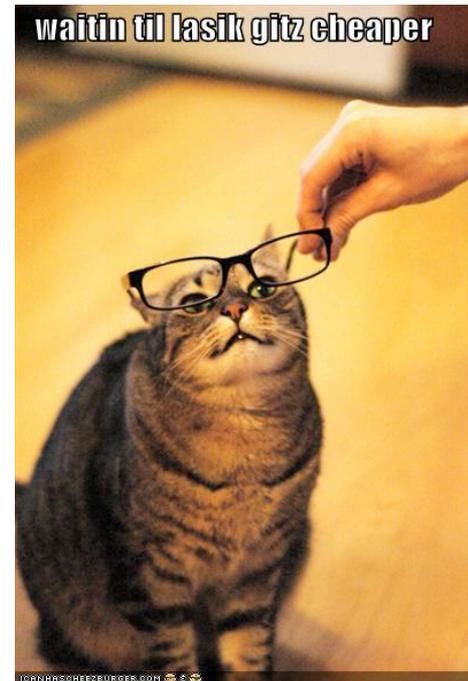
# Greedy algorithms

Build the final solution piece by piece

Being short sighted on each piece

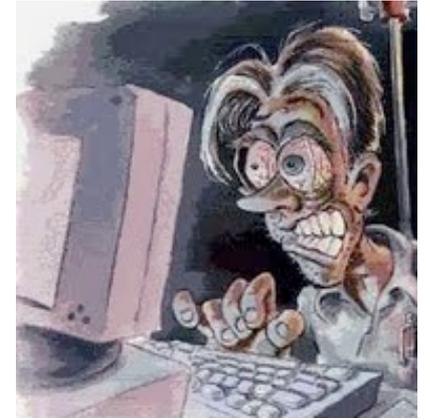
Never undo a decision

Know when you see it



# End of Semester blues

Can only do one thing at any day: what is the maximum number of tasks that you can do?



Write up a term paper

Party!

Exam study

Homework

331 HW

Project

Sunday

Monday

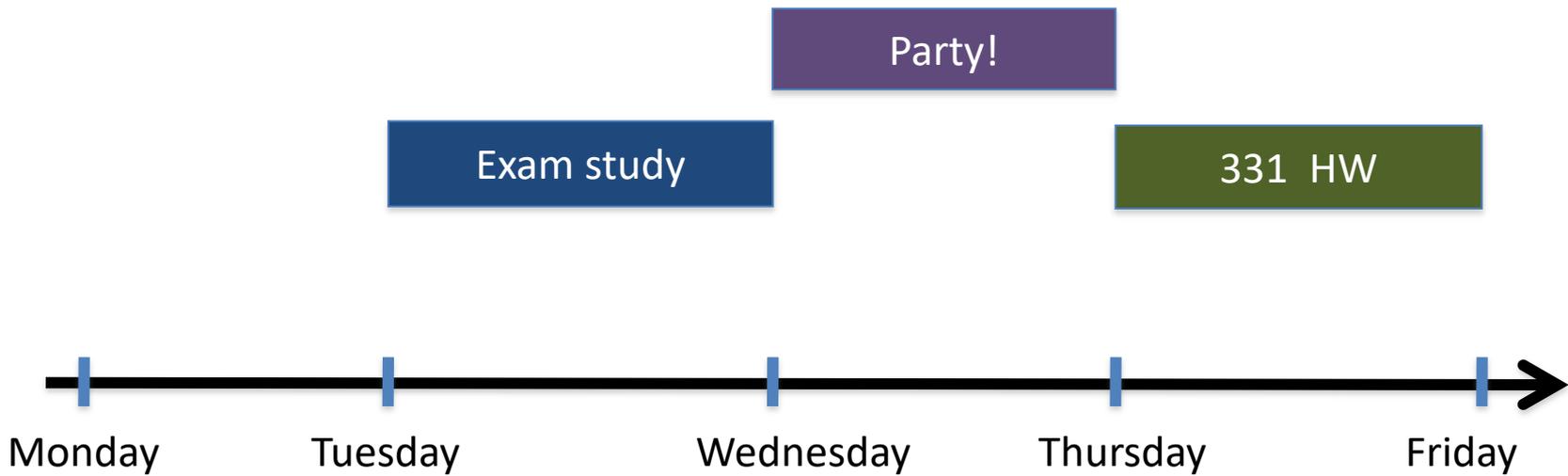
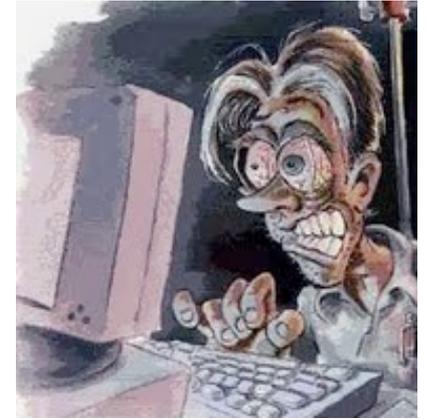
Tuesday

Wednesday

Thursday

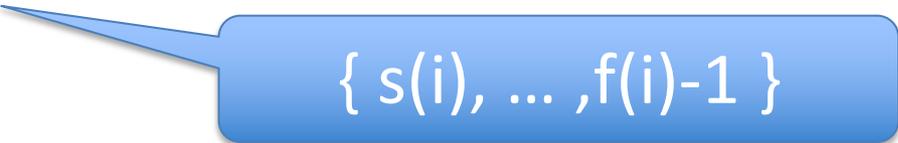
# The optimal solution

Can only do one thing at any day: what is the maximum number of tasks that you can do?



# Interval Scheduling Problem

**Input:**  $n$  intervals  $[s(i), f(i))$  for  $1 \leq i \leq n$



$\{ s(i), \dots, f(i)-1 \}$

**Output:** A *schedule*  $S$  of the  $n$  intervals

No two intervals in  $S$  conflict

$|S|$  is maximized

# Algorithm with examples

## Interval Scheduling via examples

In which we derive an algorithm that solves the Interval Scheduling problem via a sequence of examples.

### The problem

In these notes we will solve the following problem:

#### Interval Scheduling Problem

**Input:** An input of  $n$  intervals  $[s(i), f(i))$ , or in other words,  $\{s(i), \dots, f(i) - 1\}$  for  $1 \leq i \leq n$  where  $i$  represents the intervals,  $s(i)$  represents the start time, and  $f(i)$  represents the finish time.

**Output:** A schedule  $S$  of  $n$  intervals where no two intervals in  $S$  conflict, and the total number of intervals in  $S$  is maximized.

### Sample Input and Output

**Input:**

# Example 1

No intervals overlap



# Algorithm?



No intervals overlap

$R$ : set of requests

Set  $S$  to be the empty set

While  $R$  is not empty

    Choose  $i$  in  $R$

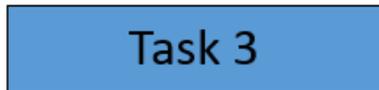
    Add  $i$  to  $S$

    Remove  $i$  from  $R$

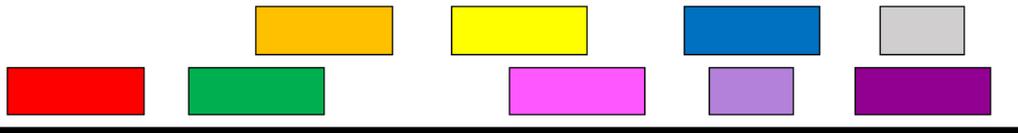
Return  $S^* = S$

# Example 2

At most one overlap



# Algorithm?



At most one overlap

$R$ : set of requests

Set  $S$  to be the empty set

While  $R$  is not empty

    Choose  $i$  in  $R$

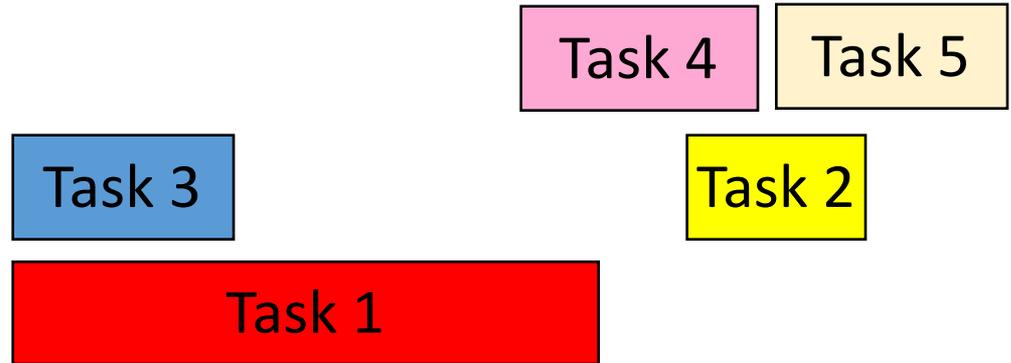
    Add  $i$  to  $S$

    Remove all tasks that conflict with  $i$  from  $R$

Return  $S^* = S$

# Example 3

More than one conflict



Set  $S$  to be the empty set

While  $R$  is not empty

    Choose  $i$  in  $R$

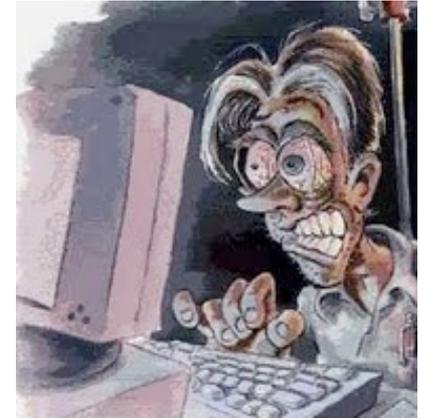
    Add  $i$  to  $S$

    Remove all tasks that conflict with  $i$  from  $R$

Return  $S^* = S$

# Greedily solve your blues!

Arrange tasks in some order and iteratively pick non-overlapping tasks



Write up a term paper

Party!

Exam study

331 HW

Project

Monday

Tuesday

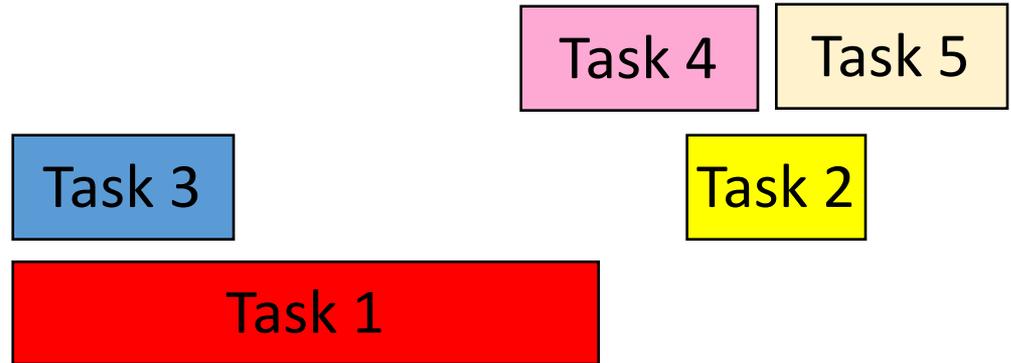
Wednesday

Thursday

Friday

# Making it more formal

More than one conflict



Set  $S$  to be the empty set

While  $R$  is not empty

**Choose**  $i$  **in**  $R$  that minimizes  $v(i)$

    Add  $i$  to  $S$

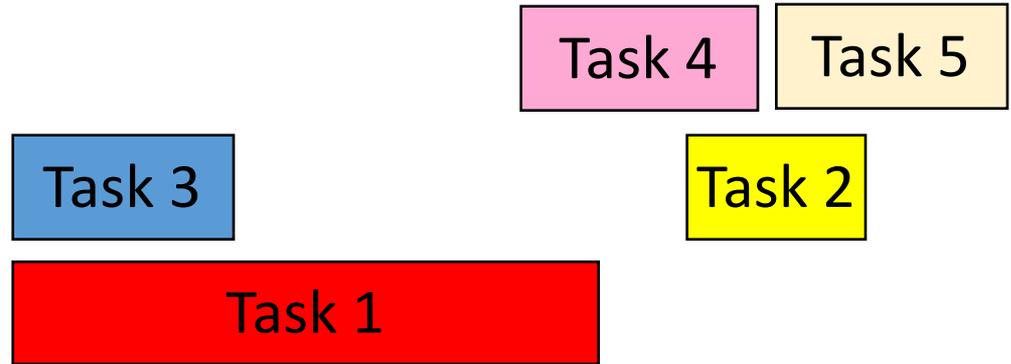
    Remove all tasks that conflict with  $i$  from  $R$

Return  $S^* = S$

Associate a  
value  $v(i)$   
with task  $i$

# What is a good choice for $v(i)$ ?

More than one conflict



Set  $S$  to be the empty set

While  $R$  is not empty

    Choose  $i$  in  $R$  that minimizes  $v(i)$

    Add  $i$  to  $S$

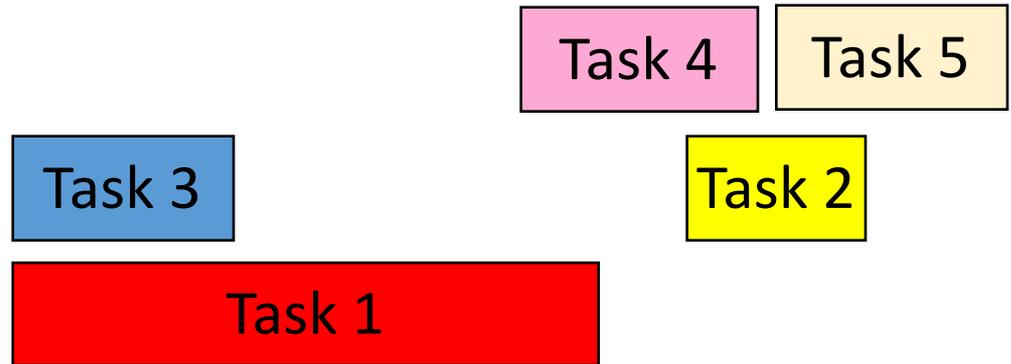
    Remove all tasks that conflict with  $i$  from  $R$

Return  $S^* = S$

Associate a  
value  $v(i)$   
with task  $i$

$$v(i) = f(i) - s(i)$$

Smallest duration first



Set  $S$  to be the empty set

While  $R$  is not empty

    Choose  $i$  in  $R$  that minimizes  $f(i) - s(i)$

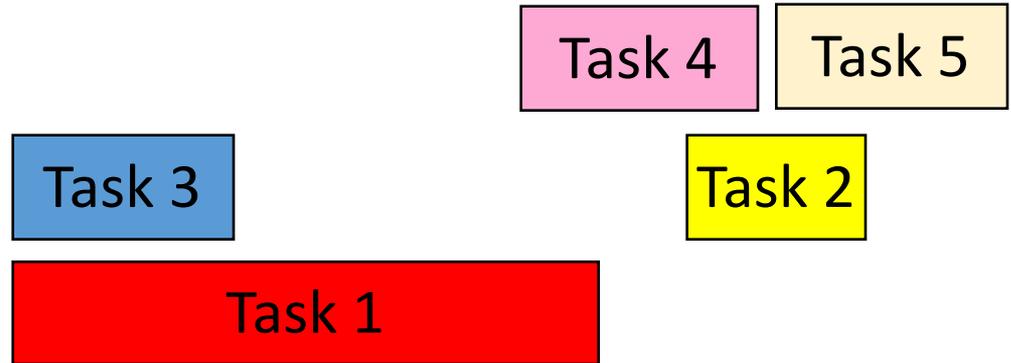
    Add  $i$  to  $S$

    Remove all tasks that conflict with  $i$  from  $R$

Return  $S^* = S$

$$v(i) = s(i)$$

Earliest time first?



Set  $S$  to be the empty set

While  $R$  is not empty

    Choose  $i$  in  $R$  that minimizes  $s(i)$

    Add  $i$  to  $S$

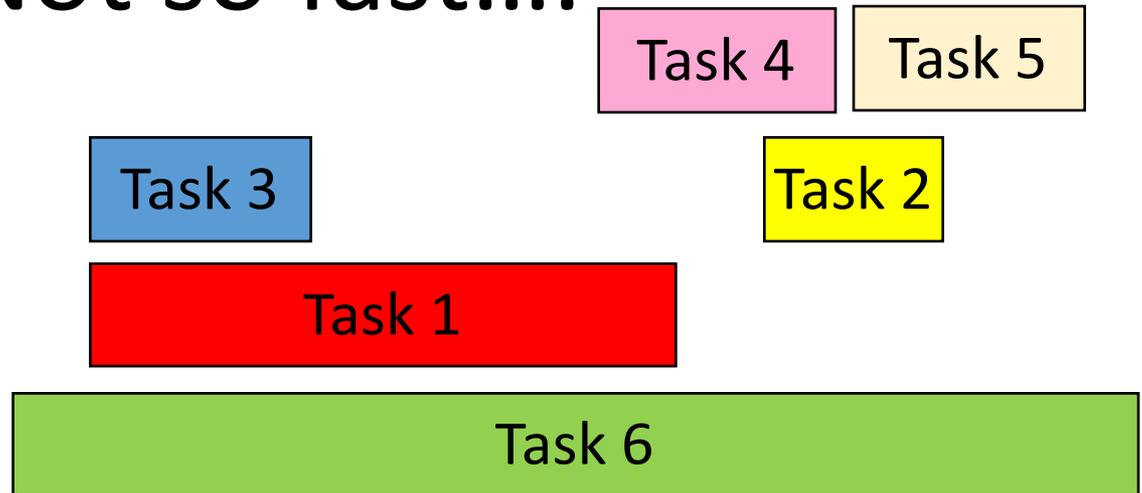
    Remove all tasks that conflict with  $i$  from  $R$

Return  $S^* = S$

So are we  
done?

# Not so fast....

Earliest time first?



Set  $S$  to be the empty set

While  $R$  is not empty

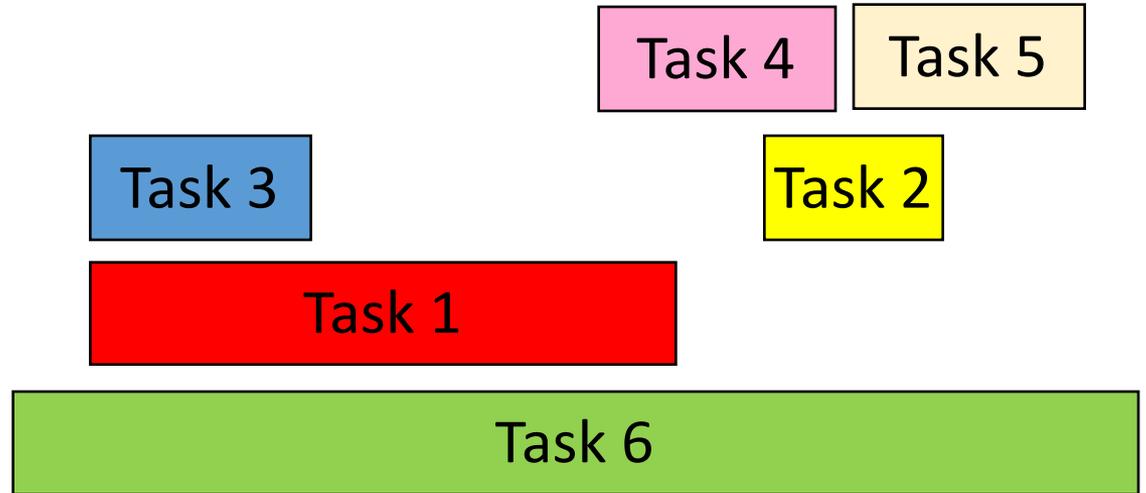
    Choose  $i$  in  $R$  that minimizes  $s(i)$

    Add  $i$  to  $S$

    Remove all tasks that conflict with  $i$  from  $R$

Return  $S^* = S$

# Pick job with minimum conflicts



Set  $S$  to be the empty set

While  $R$  is not empty

    Choose  $i$  in  $R$  that has smallest number of conflicts

    Add  $i$  to  $S$

    Remove all tasks that conflict with  $i$  from  $R$

Return  $S^* = S$

So are we  
done?

# Nope (but harder to show)

Set  $S$  to be the empty set

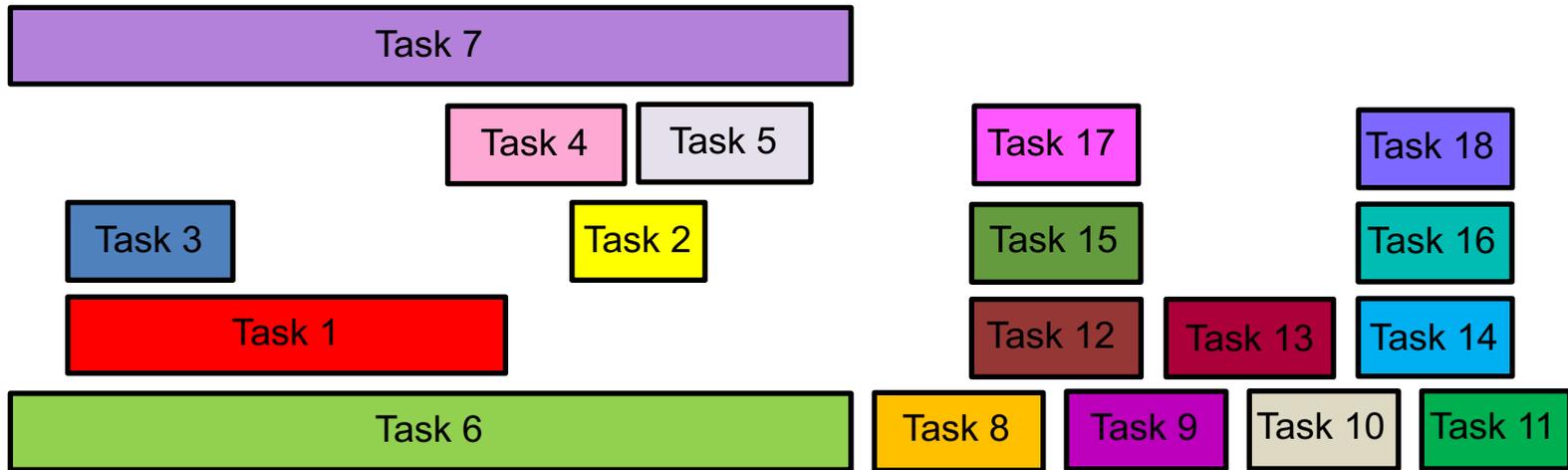
While  $R$  is not empty

    Choose  $i$  in  $R$  that has smallest number of conflicts

    Add  $i$  to  $S$

    Remove all tasks that conflict with  $i$  from  $R$

Return  $S^* = S$



Set  $S$  to be the empty set

While  $R$  is not empty

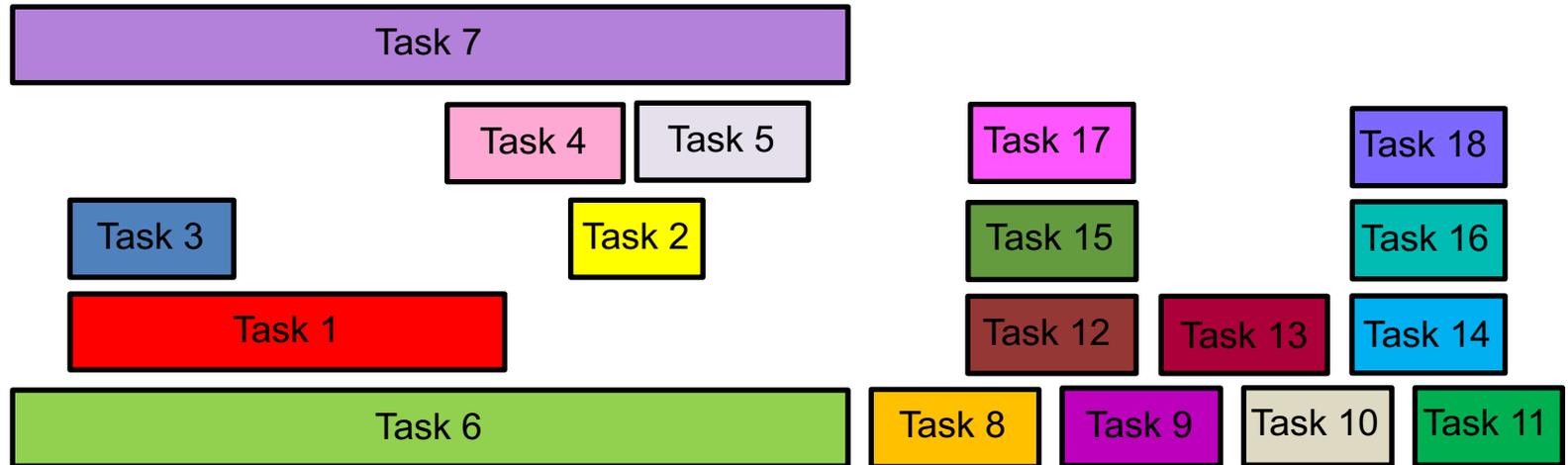
    Choose  $i$  in  $R$  that has smallest number of conflicts

    Add  $i$  to  $S$

    Remove all tasks that conflict with  $i$  from  $R$

Return  $S^* = S$

# Algorithm?



Set  $S$  to be the empty set

While  $R$  is not empty

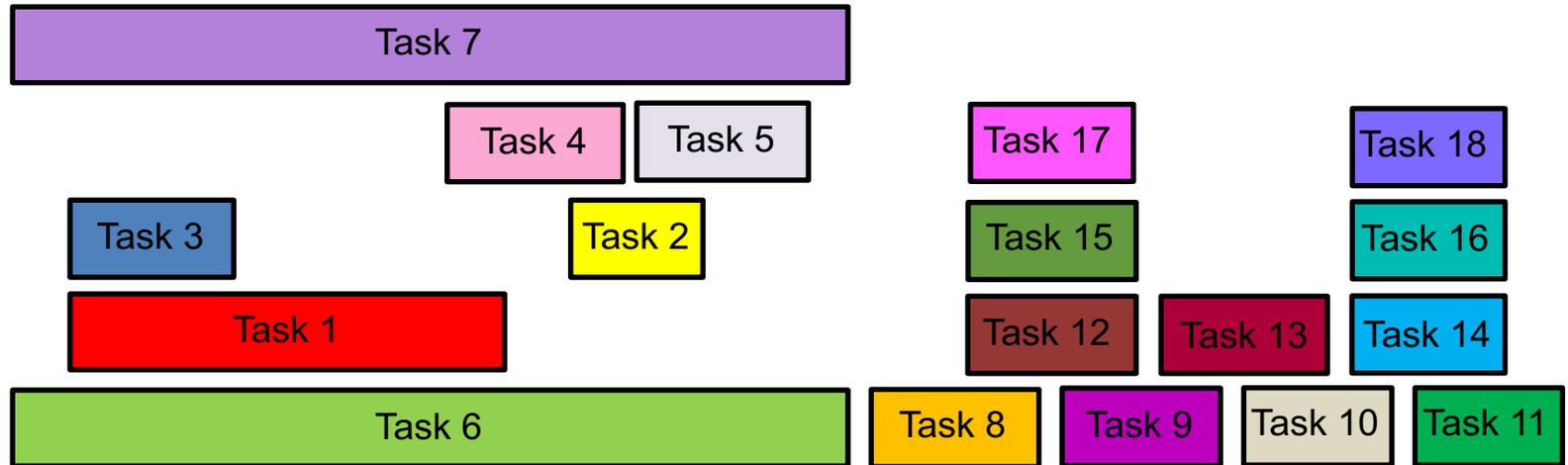
    Choose  $i$  in  $R$  that minimizes  $v(i)$

    Add  $i$  to  $S$

    Remove all tasks that conflict with  $i$  from  $R$

Return  $S^* = S$

# Earliest finish time first



Set  $S$  to be the empty set

While  $R$  is not empty

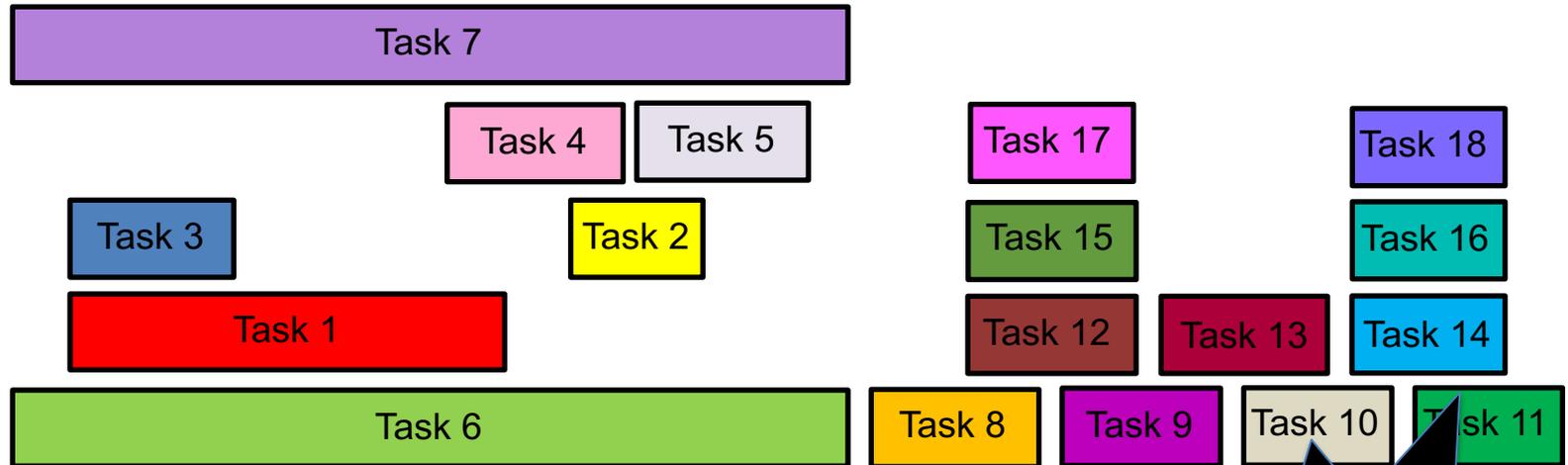
    Choose  $i$  in  $R$  that minimizes  $f(i)$

    Add  $i$  to  $S$

    Remove all tasks that conflict with  $i$  from  $R$

Return  $S^* = S$

# Find a counter-example?



Set  $S$  to be the empty set

While  $R$  is not empty

    Choose  $i$  in  $R$  that minimizes  $f(i)$

    Add  $i$  to  $S$

    Remove all tasks that conflict with  $i$  from  $R$

Return  $S^* = S$

It  
works!

# Questions?



# Today's agenda

Prove the correctness of the algorithm

# Final Algorithm

$R$ : set of requests

Set  $S$  to be the empty set

While  $R$  is not empty

    Choose  $i$  in  $R$  with the earliest finish time

    Add  $i$  to  $S$

    Remove all requests that conflict with  $i$  from  $R$

Return  $S^* = S$