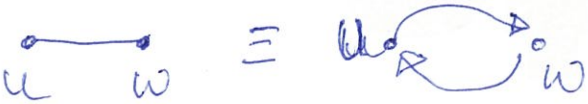
Sep 18

$G = (V, E)$ set of edges $E \subseteq V \times V$

graph; set of vertices / nodes

Default: $n = |V|$; $m = |E|$

Def: G is undirected $\iff$ $\forall\, u \neq w \in V$ $(u, w) \in E$ $\iff$ $(w, u) \in E$



o/w (otherwise) G is directed

(•) Airline map (U)

(•) Wikipedia page

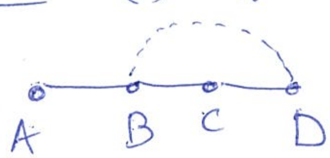
Default: G will be undirect.

Claim: Every undirect graph is also directed.

Pf idea:

Def: A path in $G = (V, E)$ is sequence of vertices $u_1, \ldots, u_k$ s.t. $\forall\, i \in [k-1]$, $(u_i, u_{i+1}) \in E$

notes: (i) u_i need not be distinct (ii) holds for directed graphs



D, C, B, A ✓

A, B, C, D ✓

A, B, C, B ✓

A, C, D ✗

D, C, B, A ✗

A, B, C, D ✓

A, B, C, B ✗

A, C, D ✗

Def: A simple path does NOT have ANY repeated vertices

By default: All paths are simple.

Def: length of a path = # edges in the path

len(A, B, C, D) = 3

Q: What is max length of a (simple) path?

A: $n-1$. Ex (Hint: use PHP)

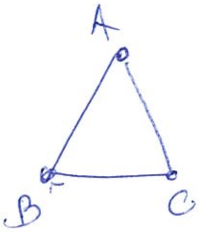
Def: A cycle $u_1, \ldots, u_k$ is a path s.t

(1) $u_1, \ldots, u_{k-1}$ are distinct (2) $u_1 = u_k$

(3) (•) G is undirected: $k \geq 4$ [ A, B, A)

(•) G is directed $k \geq 3$

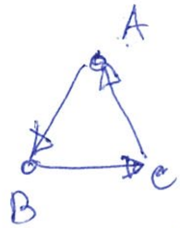
(B, C, B) ✓



A, C, B, A ✓

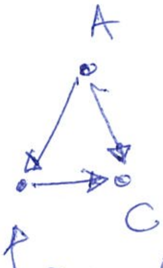
A, B, C, A ✓

"triangle"



A, C, B, A X

A, B, C, A ✓



A, C, B, A X

A, B, C, A X

Directed Acyclic Graph (DAG)

Def: u & w are connected (undirected G) ∃ u–w path

_______ (strongly) connected (directed graphs) ∃ u – w path, w – u path

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Def: A (directed) graph is (strongly) connected if $\forall u \neq w$, u & w are (strongly) connected.