PROP: Let T be a BFS tree for $G = (U, E)$

If $(u, w) \in E$ s.t. $u \in L_i \wedge w \in L_j$

$$\Rightarrow |i - j| \leq 1 \quad \Longleftrightarrow \quad i \in \{j-1, j, j+1\}$$

Pf idea WLOG assume $i \leq j$ [o/w switch $i \& j$]

without loss of generality
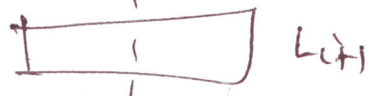
for contradiction assume $|i - j| > 1 \Rightarrow j > i+1$

$$\Longleftrightarrow j \geq i+2$$

[S] L_0

Consider situation when BFS was creating L_{i+1}

$u_\bullet$ L_i $\Rightarrow u \in L_i$, $w \in L_0, \dots, L_{i+1}$

L_{i+1} $\rightarrow (u, w) \in E$

$\Rightarrow$ By BFS alg $w \in L_{i+1}$

$\bullet w$ L_j $\Rightarrow$ contradicts $w \in L_j$ for $j > i+1$