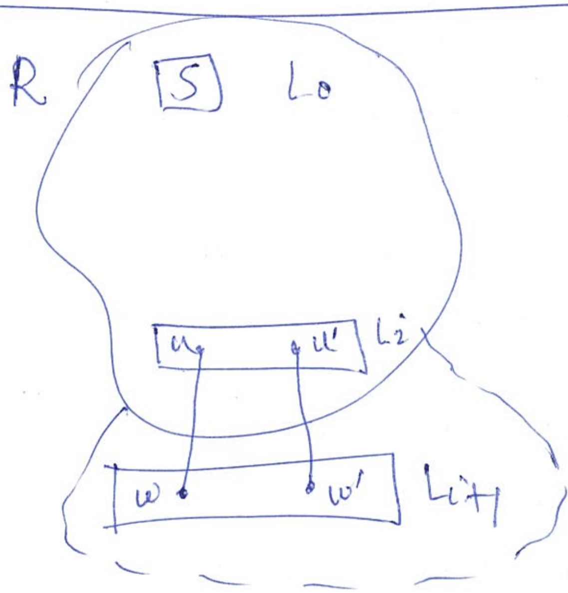
Sep 23

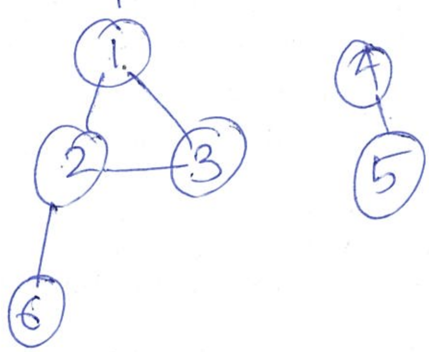
Explore (s)

0. $R = \{s\}$
1. While $\exists\ (u,w) \in E$ s.t. $u \in R$ and $w \notin R$ (s.t. = such that)
   Add w to R.
2. Output $R^* = R$

Lem 0: Explore always terminates



Def: Set of all vertices connected to s is called it's connected component $CC(s)$



$CC(2) = \{1, 2, 3, 6\}$

$CC(5) = \{4, 5\}$

THEOREM: for all G, start vertices s, $R^* = CC(s)$

General trick: to show $A = B \iff A \subseteq B$ and $B \subseteq A$

Lemma 1: $R^* \subseteq CC(s)$

Lemma 2: $CC(s) \subseteq R^*$

Lemma 1 + Lemma 2 $\Rightarrow$ THM.

THM $\Rightarrow$ BFS is correct.

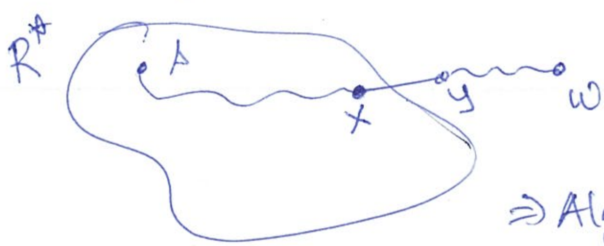
Ex: By induction

Pf idea of Lemma 2:

Pf. by contradiction

Assume $CC(s) \not\subseteq R^* \Rightarrow \exists\ w \in CC(s)$ BUT $w \notin R^*$

$\iff \exists$ s-w path P in G but $w \notin R^*$
[$w \in CC(s)$]



Since P starts inside of R^* but ends up outside of R^*,

$\Rightarrow \exists\ (x, y) \in P$ s.t. $x \in R^*$ and $y \notin R^*$

$\Rightarrow$ y should have been added to R by Explore

$\Rightarrow$ Algo has not terminated $\Rightarrow$ contradicts the existence of R^*