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Dijkstra's algo

$d'(w) = \min_{\substack{u \in R \\ (u,w) \in E}} \{ d(u) + \ell_{(u,w)} \}$

0. $R = \{s\}$, $d(s) = 0$

1. While $\exists x \notin R$ s.t $\exists u \in R$ with $(u,x) \in E$
   Pick w among all such x's with smallest $d'(w)$ value
   Add w to R
   $d(w) = d'(w)$

Def: Let P_u be the s-u path in "Dijkstra tree"

THM: $\forall u \in V$, P_u is a shortest s-u path.

$\xRightarrow{\text{(Ex.)}}$ $d(u)$ are computed correctly $\Rightarrow$ Dijkstra is correct

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Lemma 1: At the end of each iteration of while loop, $\forall u \in R$, P_u is a shortest s-t path.

Lemma 2: $u \in V$ s.t $\exists$ s-u path $\Leftrightarrow$ $u \in R$ at the end.
(Ex)

Lemmas 1+2 $\Rightarrow$ THM.

Proof (idea) of Lemma 1: By induction on $|R|$

Base case: $|R| = 1$, $R = \{s\}$, $d(s) = 0$ ✓

I.H. Assume is true for $|R| = k$ $(k \geq 1)$

I.S: Argue for $|R| = k+1$