Nov 6

<u>Simplified problem</u>: Only compute the <u>value</u> of an optimal schedule.

$0 \le j \le n$

$OPT(j)$ = value of ~~the~~ an optimal solution $\underbrace{[j]}_{\{(s_1,f_1,v_1),\ldots,(s_j,f_j,v_j)\}}$

<u>Goal:</u> Compute $OPT(n)$.

<u>Def:</u> $\mathcal{O}_j$ be an optimal solution for $[j]$

$$OPT(j) \underset{\text{by definition of } OPT(j) \text{ \& } \mathcal{O}_j}{=} v(\mathcal{O}_j)$$

$OPT(0) = 0$

<u>Goal:</u> $OPT(j) = \max\{OPT(j-1),\ v_j + OPT(\underbrace{p(j)}_{<j})\}$

TBA

<u>Case 1:</u> $j \notin \mathcal{O}_j$ <u>Claim 1:</u> $\mathcal{O}_j$ is also optimal for $[j-1]$

$$\Rightarrow OPT(j) \underset{\text{def of } \mathcal{O}_j}{=} v(\mathcal{O}_j) \underset{\text{Claim 1}}{=} OPT(j-1)$$

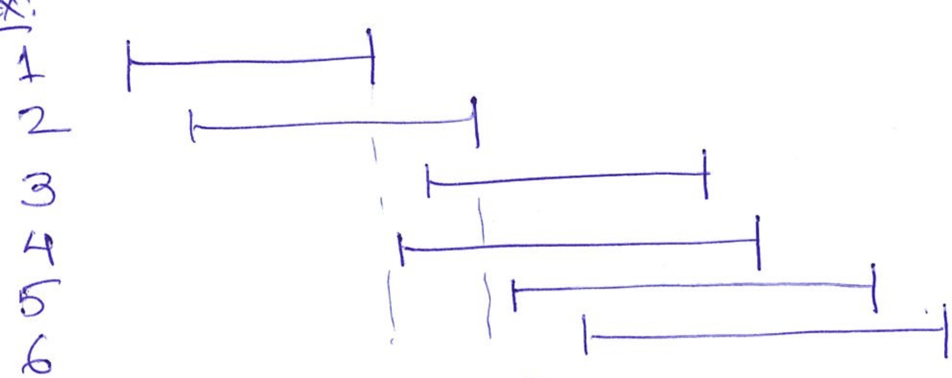
$\mathcal{O}_j$ is NOT optimal for $[j-1]$

<u>Pf (idea) of Claim 1:</u> Assume $\exists$ a schedule $\mathcal{O}'$ for $[j-1]$ s.t. $v(\mathcal{O}') > v(\mathcal{O}_j)$

But $\mathcal{O}'$ is also valid for $[j] \Rightarrow$ contradicts the assumption that $\mathcal{O}_j$ is optimal for $[j]$.

<u>Case 2:</u> $j \in \mathcal{O}_j$ | <u>Def</u>: $p(j)$ = largest $i < j$ s.t. i & j do not conflict
$= 0$ if no such i $\exists$.

<u>Ex:</u>



$OPT(6) = \max\{OPT(5),\ v_6 + OPT(2)\}$

$p(1) = 0$
$p(2) = 0$
$p(3) = 1$
$p(4) = 1$
$p(5) = 2$
$p(6) = 2$

① $p(j)+1, \ldots, j-1$ conflict with j

② $1, \ldots, p(j)$ will NOT conflict with j

Case 2: $j \in O_j$ Claim 2: $O_j \setminus \{j\}$ is an optimal solution for $[p(j)]$

$$\Rightarrow OPT(j) \underset{\substack{\uparrow \\ \text{def of } O_j}}{=} v(O_j) = \cancel{v(O)}\, v_j + v(O_j \setminus \{j\})$$

$$\underset{\substack{\uparrow \\ \text{claim 2}}}{=} v_j + OPT(p(j))$$

Pf (idea) of Claim 2: Assume $O_j \setminus \{j\}$ is NOT optimal for $[p(j)]$

$\Rightarrow \exists\, O'$ which is a valid schedule for $[p(j)]$ & $v(O') > v(O_j \setminus \{j\})$

Note: $O' \cup \{j\}$ is a valid schedule for $[j]$

but $v(O' \cup \{j\}) = v(O') + v_j > v(O_j \setminus \{j\}) + v_j = v(O_j)$

$\Rightarrow$ contradicts optimality of O_j for $[j]$.

Ex: Compute $p(j)$ for $1 \le j \le n$ in $O(n \log n)$ time.

Bonus Ex: $\Omega(n \log n)$ comparisons to compute $p(j)$ for all j.