Nov 22

SAT formula: AND (or conjunction) of Clauses

$C_1, \dots, C_m$ $\qquad X = \{x_1, \dots, x_n\}$ ← vars

$\equiv C_1 \wedge C_2 \wedge \dots \wedge C_m$.

Clause: OR (or disjunction) of literals

$t_1 \vee t_2 \vee \dots \vee t_e$ $\qquad$ Each $z_i \in \{x_1, \dots, x_n, \bar{x}_1, \dots, \bar{x}_n\}$

Eg: $(x_1 \vee \bar{x}_2), (\bar{x}_1 \vee \bar{x}_3), (x_2 \vee \bar{x}_3)$ —— (*)

$m = 3$

Assignment: $v: X \to \{0,1\}$
(2^n possible assignments)

$x_1 = 0$	1	0
$x_2 = 0$	1	0
$x_3 = 0$	1	1

An assignment satisfies a formula if the formula evaluates to true under the assignment

$\Leftrightarrow \equiv$ the assignment satisfies ALL clauses.

→ An assignment satisfies a clause if the clause evaluates to 1 under the assignment.

Assignment: $(0,0,0) \Rightarrow (0,0,0)$ satisfies (*)

$(x_1 \vee \bar{x}_2) \;\underline{\underline{\downarrow}}\; 0 \vee \bar{0} = 0 \vee 1 = 1$

$(\bar{x}_1 \vee \bar{x}_3) = \bar{0} \vee \bar{0} = 1 \vee 1 = 1$

$(x_2 \vee \bar{x}_3) = 0 \vee \bar{0} = 0 \vee 1 = 1$

for X $(1,1,1) = (1 \vee \bar{1}) \wedge (\bar{1} \vee \bar{1}) \wedge (1 \vee \bar{1}) = 0$

X $(0,0,1) = (0 \vee \bar{0}) \wedge (\bar{0} \vee \bar{1}) \wedge (0 \vee \bar{1}) = 0$

Q: Given a SAT formula does it have a satisfying assignment?

3-SAT problem: Same as the SAT problem with the extra restriction that each clause has EXACTLY 3 literals.

THM: 3-SAT $\leq_p$ Independent set.

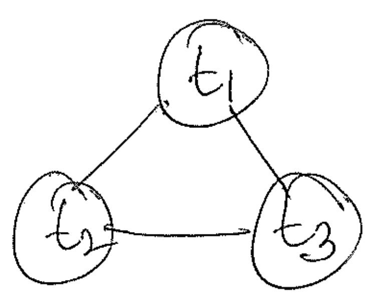
→ Use "gadgets"

2 equivalent ways of looking at 3-SAT:

→ Make independent 0/1 choices for $x_1, \dots, x_n$ s.t you satisfy at least one literal in each clause.

→ Pick one literal for each clause s.t all picked literals do not conflict ← you do NOT pick both x_i & $\overline{x_i}$

gadget: $C = t_1 \vee t_2 \vee t_3$



Each of the 3 IS $\{t_1\}$ or $\{t_2\}$ or $\{t_3\}$ correspond to which literal we'll choose from clause C.

Redux: Given a 3SAT formula: $C_1, \dots, C_m$

Claim/design: $\longmapsto (G, m)$ s.t 3SAT formula is satisfiable $\iff$ G has an IS of size $\geq m$.

(G, k) for IS

Claim: Done if we can compute such G in poly time.

Redux!

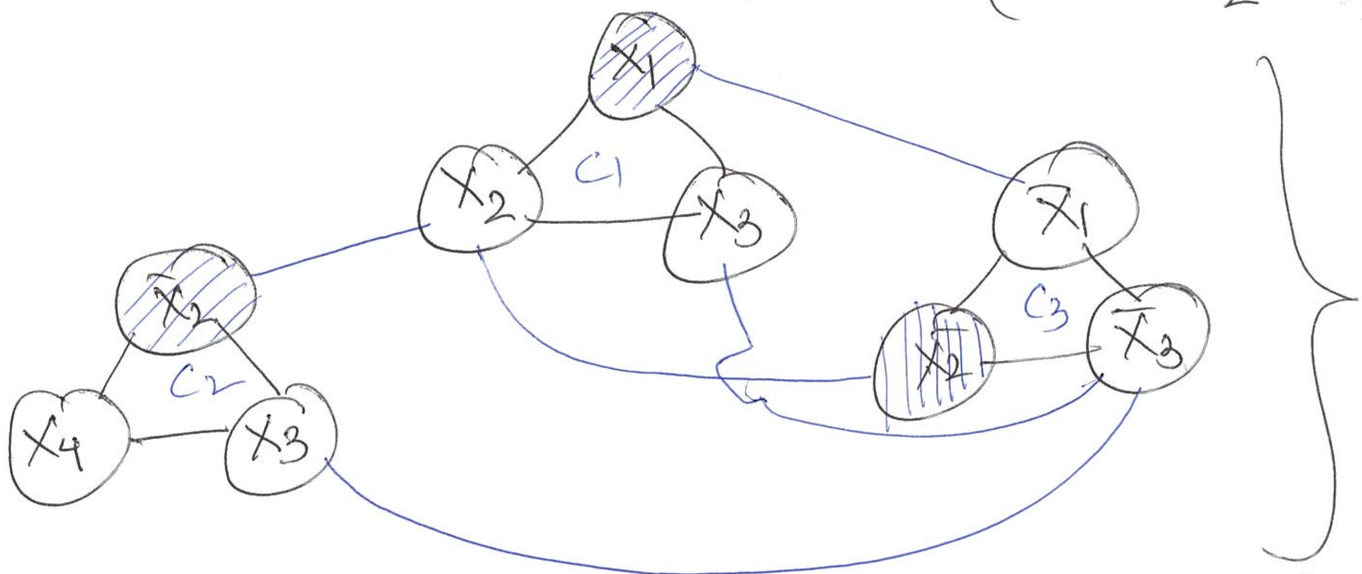
Step 1: Replace each clause C_i by it's "triangle"

Step 2: Add an edge between x_i & $\bar{x}_i$ if they occur in the formula

$(x_1 \vee x_2 \vee x_3), = C_1$

$(\bar{x}_2 \vee x_3 \vee x_4), = C_2$

$(\bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3), = C_3$



G

To argue: 3-SAT formula is ~~satf~~ satisfiable

$\Longleftrightarrow$ G has an IS of size $\cancel{\geq} = m$.

Since all IS in G have size $\leq m$. (by construction)

Pf sketch:

$\Rightarrow$: Given SAT formula that is satisfiable

$v: X \to \{0,1\}$

Ex: $x_1 = 1$, $x_2 = 0$, $x_3 = 1$, $x_4 = 1$

$\Rightarrow$ G has an IS of size m

$\Rightarrow$ each clause has at least one true literal

$\Rightarrow$ pick any one

Claim: The ~~liter~~ m literals that we pick form an IS in G.