THEOREM: For any input $(M, W, 2n$ pref list$)$ the GS algo outputs a stable matching.

$\Rightarrow$ Every input has a stable matching.

LEMMA 1: For every input, the GS algo terminates in $\leq n^2$ iterations.

LEMMA 2: The output of GS algo (S) is a perfect matching.

LEMMA 3: S has no instability

Lemmas $1+2+3 \Rightarrow$ Theorem

Pf idea Lemma 1: In each iteration, a new proposal (from w to m) is made.

$\Rightarrow$ # Iterations = #proposals $\leq$ #pairs $(w,m) = |W \times M|$
$= |W| \cdot |M| = n \cdot$
$= n^2$

Obs0: S is a matching.

Obs1: Once a man gets engaged, he keeps getting engaged to better women.

Ob2: If w proposes to m after $m' \Rightarrow m' > m$ in L_w

Lemma 4: If at the end of an iteration, w is free $\Rightarrow$ w has NOT proposed to all men.

Pf of Lemma 2: (Pf idea) Pf. by contradiction (use Obs 0, Lemmas 1 + 4, algo def.)

Pf details: Assume S is NOT a perfect matching
$\Rightarrow \exists$ a free woman w
$\underset{(\text{Lemma 4})}{\overset{(\text{Obs 0 + Algo def.})}{\Longrightarrow}} \exists$ a man m that w has not proposed to $(*)$

By Lemma 1, algo has terminated $\implies$ all free women
Algo defn have proposed to
ALL men

$\implies$ contradicts (*) 矛