Lecture 26

CSE 331

Nov 3, 2021

Please have a face mask on

Masking requirement

UR requires all students, employees and visitors – regardless of their vaccination status – to wear face coverings while inside campus buildings.

https://www.buffalo.edu/coronavirus/health-and-safety/health-safety-guidelines.html

Coding P2 due Friday

Fri, Nov 5	Kickass Property Lemma F19 F18 F17 x^2	[KT, Sec 5.4] **(Project (Problem 2 `Coding`) in)**
Mon, Nov 8	Weighted Interval Scheduling F19 F17 x^2	[KT, Sec 6.1] **(Project (Problem 2 `Reflection`) in)**
Wed, Nov 10	Recursive algorithm for weighted interval scheduling problem F19 F17 x^2	[KT, Sec 6.1] **(HW 6 out)**
Fri, Nov 12	Subset sum problem F19 F18 F17 x^2	[KT, Sec 6.1, 6.2, 6.4]
Mon, Nov 15	Dynamic program for subset sum F19 F18 F17 x^2	[KT, Sec 6.4]
Wed, Nov 17	Shortest path problem F19 F18 F17 x^2	[KT, Sec 6.8] **(HW 7 out, HW 6 in)**
Fri, Nov 19	Bellman-Ford algorithm F19 F18 F17 x^2	[KT, Sec 6.8]
Mon, Nov 22	The P vs. NP problem F19	[KT, Sec 8.1]
Wed, Nov 24	**No class**	Fall Recess
Fri, Nov 26	**No class**	Fall Recess
Mon, Nov 29	More on reductions F19	[KT, Sec 8.1]
Wed, Dec 1	The SAT problem F19	[KT, Sec 8.2] **(HW 8 out, HW 7 in)**
Fri, Dec 3	NP-Completeness F19	[KT, Sec. 8.3, 8.4] **(Project (Problem 3 `Coding`) in)**
Mon, Dec 6	k-coloring problem F19	[KT, Sec 8.7] **(Quiz 2)** **(Project (Problem 3 `Reflection`) in)**

Group formation instructions

Autolab group submission for CSE 331 Project

The lowdown on submitting your project (especially the coding and reflection) problems as a group on Autolab.

Follow instructions **EXACTLY** as they are stated

The instruction below are for Coding Problem 1

You will have to repeat the instructions below for EACH coding AND reflection problem on project on Autolab (with the appropriate changes to the actual problem).

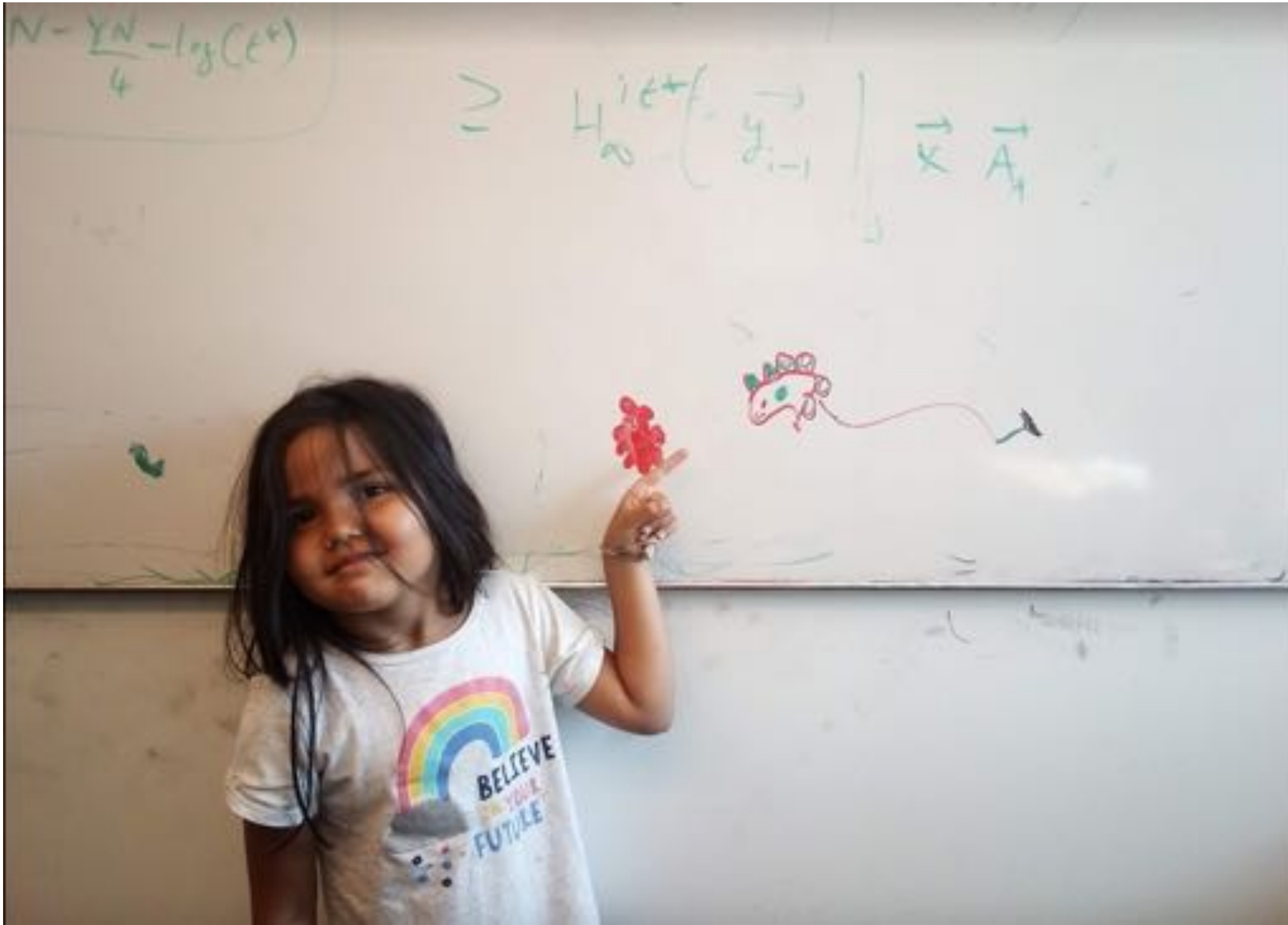
Form your group on Autolab

Groups on Autolab will NOT be automatically created

You will have to form a group on Autolab by yourself (as a group). Read on for instructions on how to go about this.

Click to add notes

Questions/Comments?



Multiplying two numbers

Given two numbers a and b in binary

$a=(a_{n-1},..,a_0)$ and $b = (b_{n-1},...,b_0)$

Compute $c = a \times b$

Elementary school algorithm is $O(n^2)$

The current algorithm scheme

Shift by O(n) bits

Adding O(n) bit numbe

Mult over n bits

$$a \bullet b = a^1b^1 \bullet 2^{2[n/2]} + (a^1b^0 + a^0b^1) \bullet 2^{[n/2]} + a^0b^0$$

Multiplication over n/2 bit inputs

$T(n) \leq 4T(n/2) + cn$

$T(1) \leq c$

T(n) is $O(n^2)$

The key identity

$$a^1b^0+a^0b^1= (a^1+a^0)(b^1+b^0) - a^1b^1 - a^0b^0$$

The final algorithm

Input: $a = (a_{n-1},..,a_0)$ and $b = (b_{n-1},...,b_0)$

Mult (a, b)

If $n = 1$ return $a_0 b_0$

$a^1 = a_{n-1},...,a_{[n/2]}$ and $a^0 = a_{[n/2]-1},..., a_0$

Compute b^1 and b^0 from b

$x = a^1 + a^0$ and $y = b^1 + b^0$

Let $p =$ Mult (x, y), $D =$ Mult (a^1, b^1), $E =$ Mult (a^0, b^0)

$F = p - D - E$

return $D \bullet 2^{2[n/2]} + F \bullet 2^{[n/2]} + E$

$T(1) \leq c$

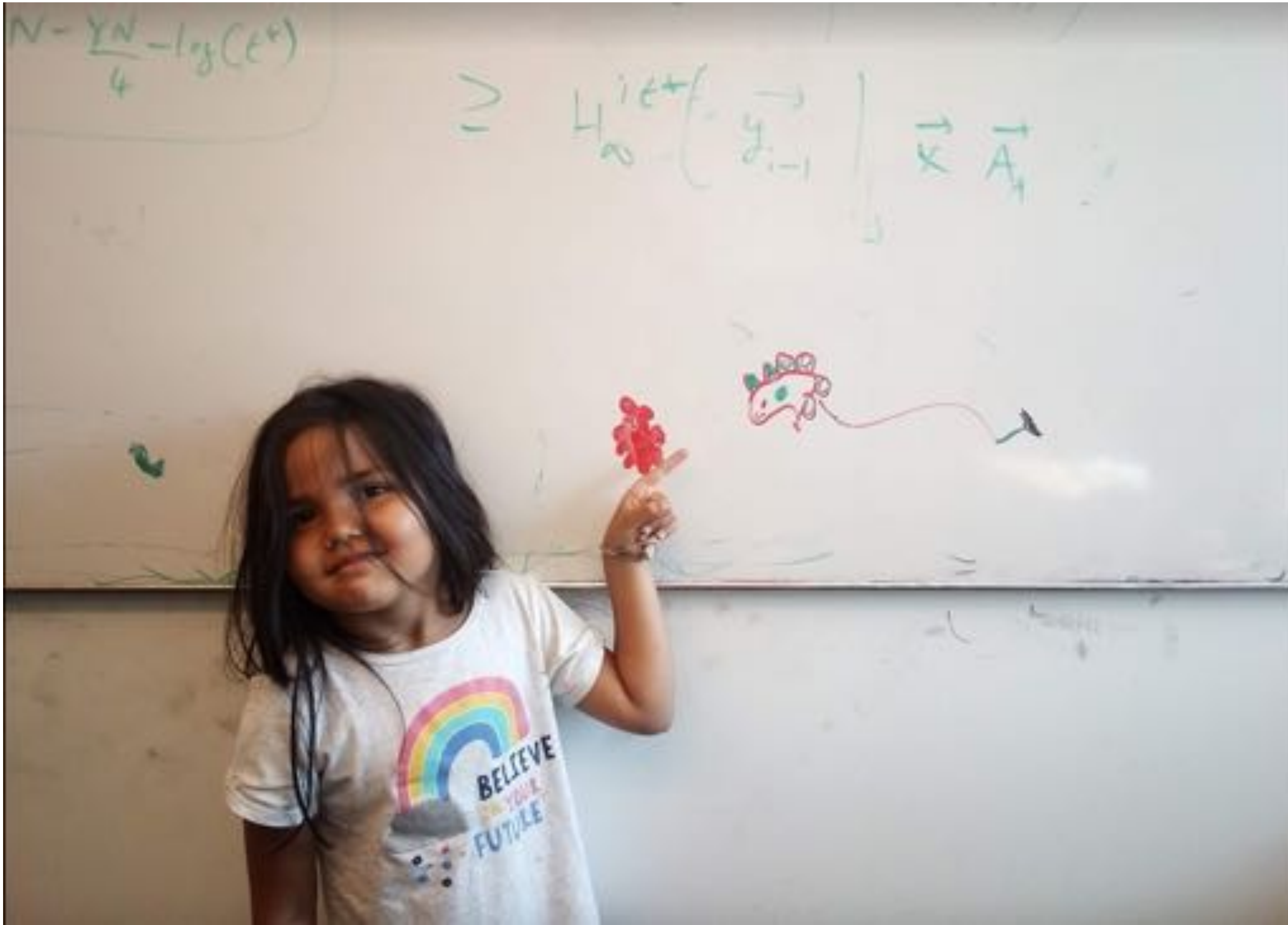
$T(n) \leq 3T(n/2) + cn$

$O(n^{\log_2 3}) = O(n^{1.59})$
run time

All green operations are $O(n)$ time

$$a \bullet b = a^1 b^1 \bullet 2^{2[n/2]} + ((a^1+a^0)(b^1+b^0) - a^1 b^1 - a^0 b^0) \bullet 2^{[n/2]} + a^0 b^0$$

Questions/Comments?

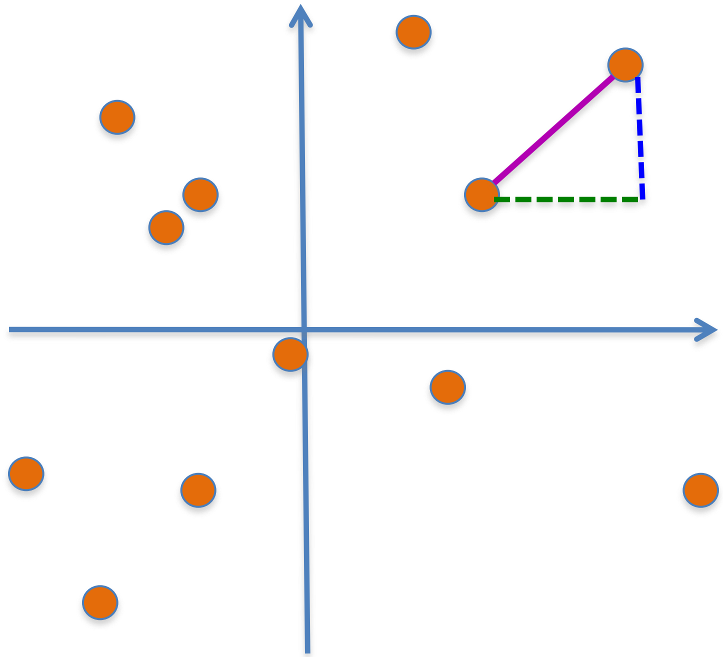


Closest pairs of points

Input: n 2-D points $P = \{p_1,\ldots,p_n\}$; $p_i=(x_i,y_i)$

$d(p_i,p_j) = ((x_i-x_j)^2+(y_i-y_j)^2)^{1/2}$

Output: Points p and q that are closest



Group Talk time

$O(n^2)$ time algorithm?

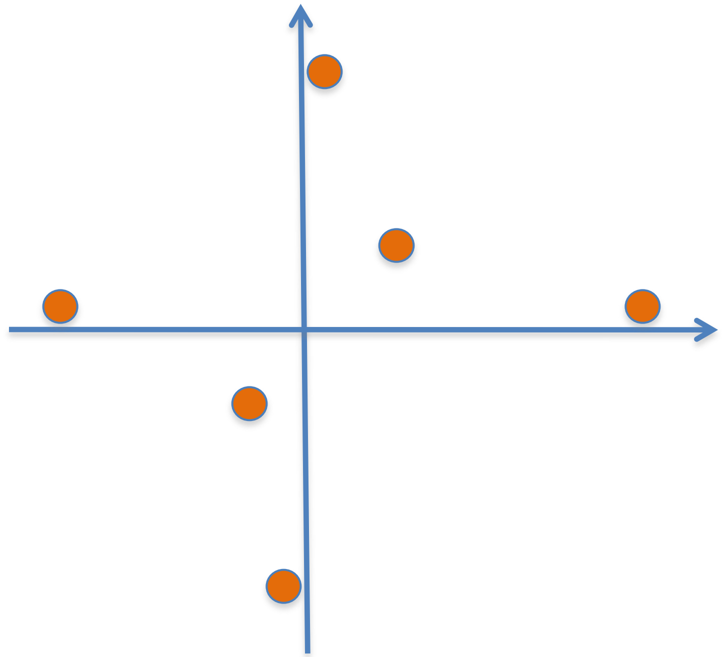
1-D problem in time $O(n \log n)$?

Sorting to rescue in 2-D?

Pick pairs of points closest in x co-ordinate

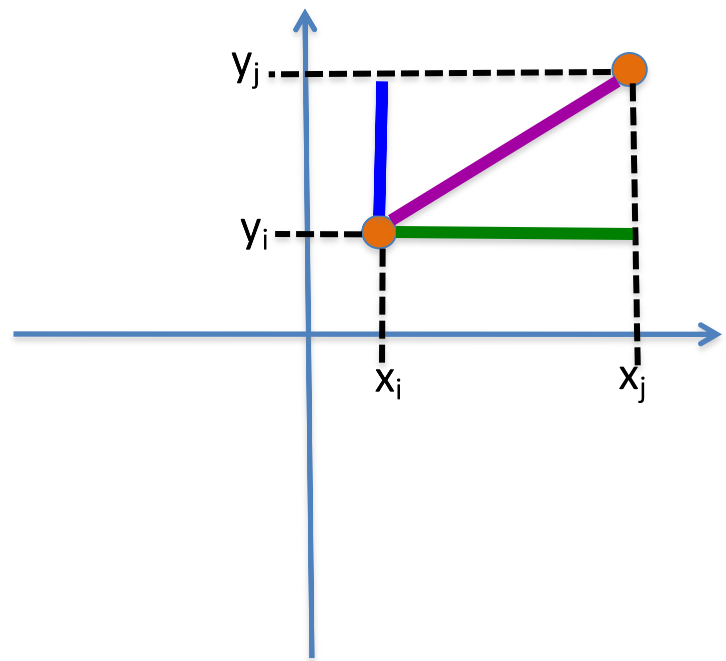
Pick pairs of points closest in y co-ordinate

Choose the better of the two



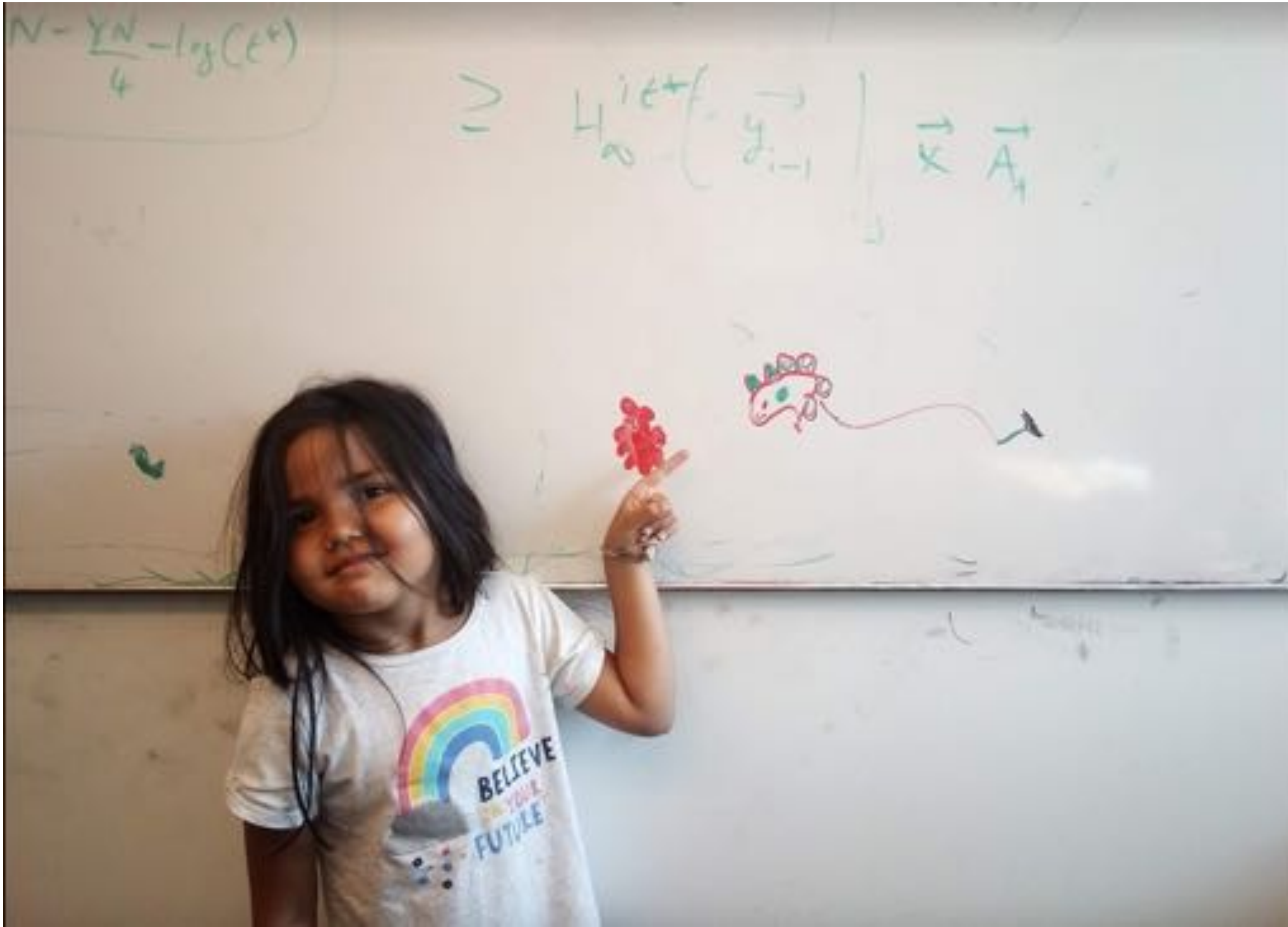
A property of Euclidean distance

$d(p_i,p_j) = ((x_i-x_j)^2+(y_i-y_j)^2)^{1/2}$



The distance is larger than the **x** or **y**-coord difference

Questions/Comments?

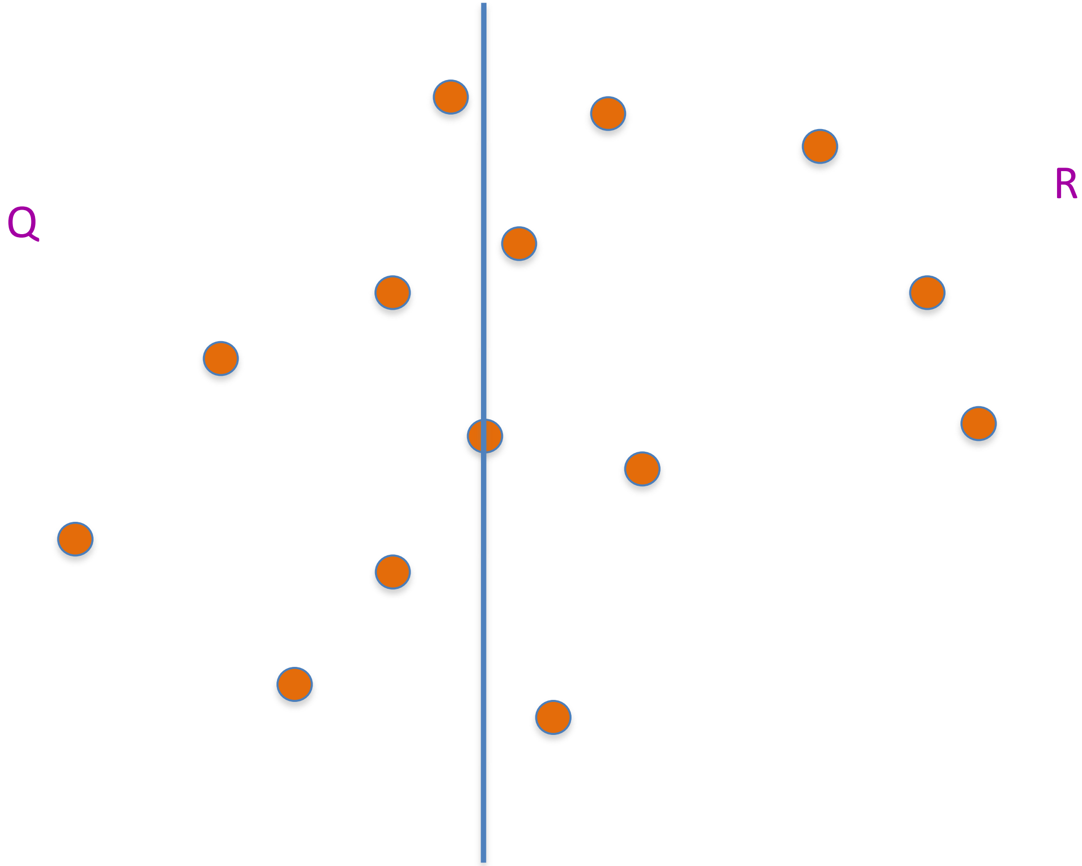


Problem definition on the board…

Rest of Today’s agenda

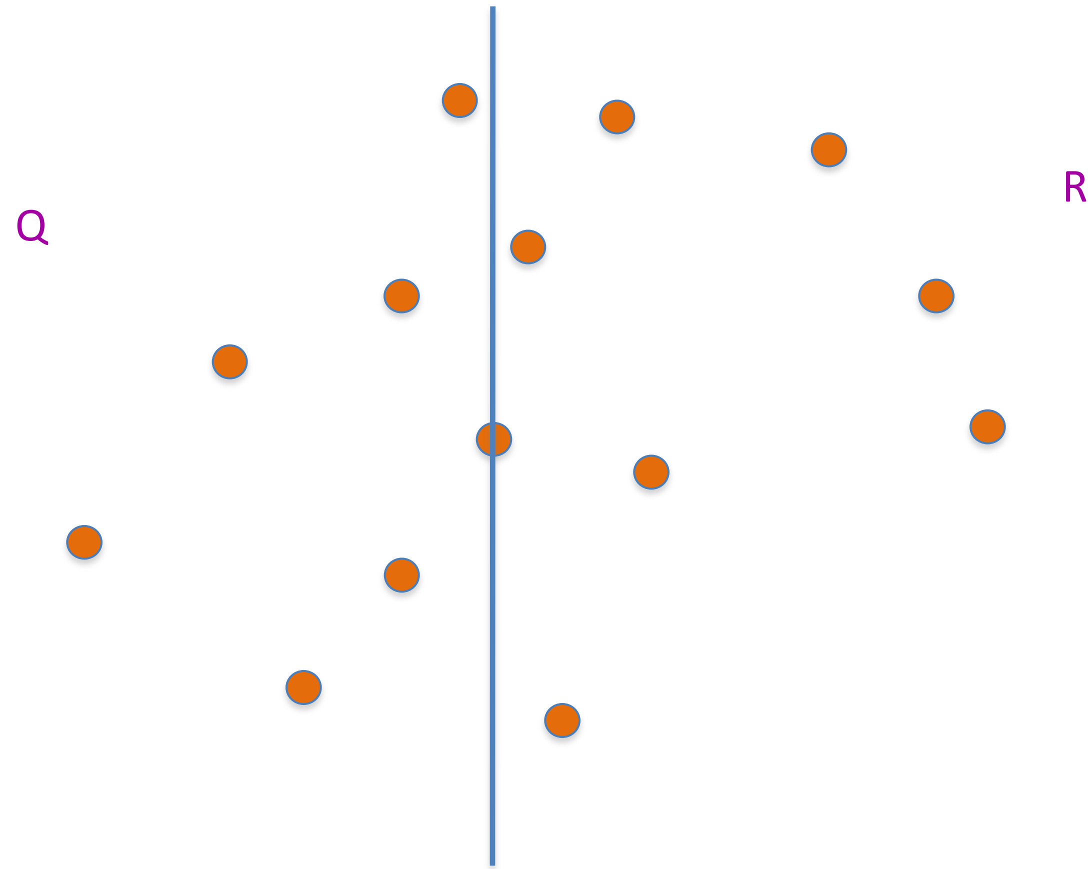
Divide and Conquer based algorithm

Dividing up P



First n/2 points according to the x-coord

Recursively find closest pairs



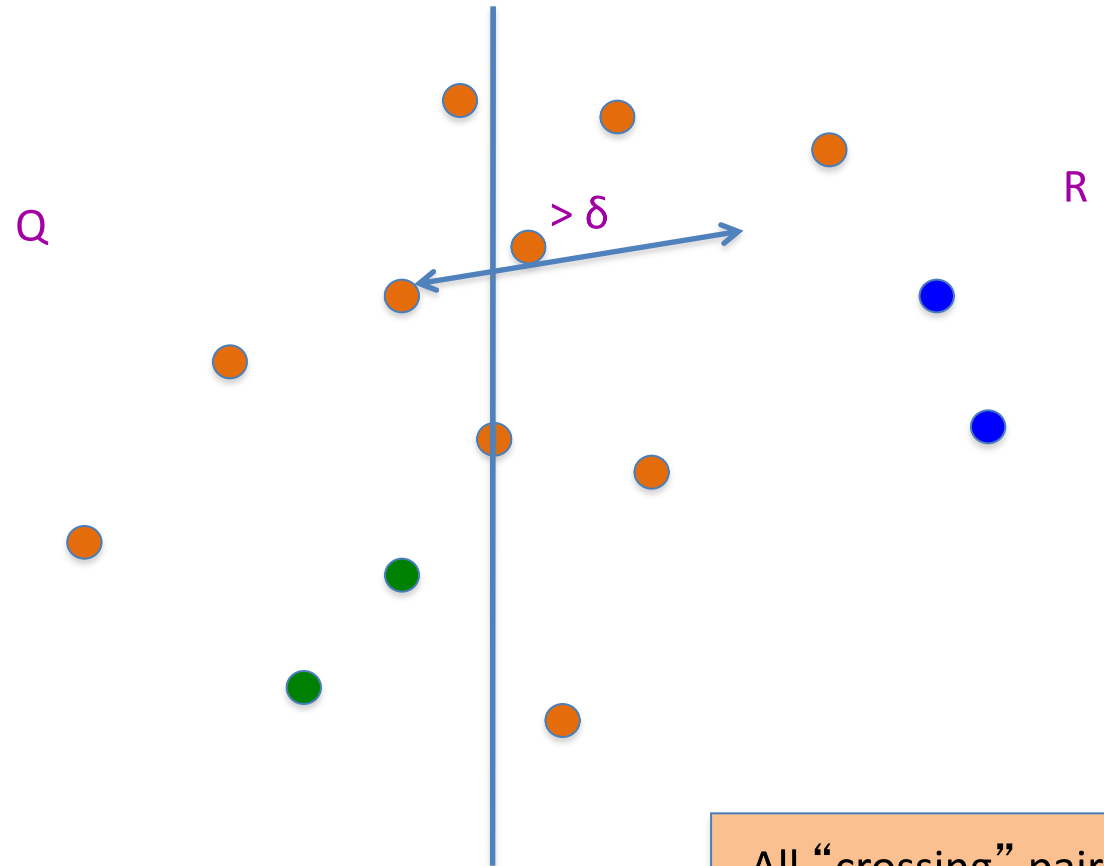
δ = min (**blue**, **green**)

An aside: maintain sorted lists

P_x and P_y are P sorted by x-coord and y-coord

Q_x, Q_y, R_x, R_y can be computed from P_x and P_y in $O(n)$ time

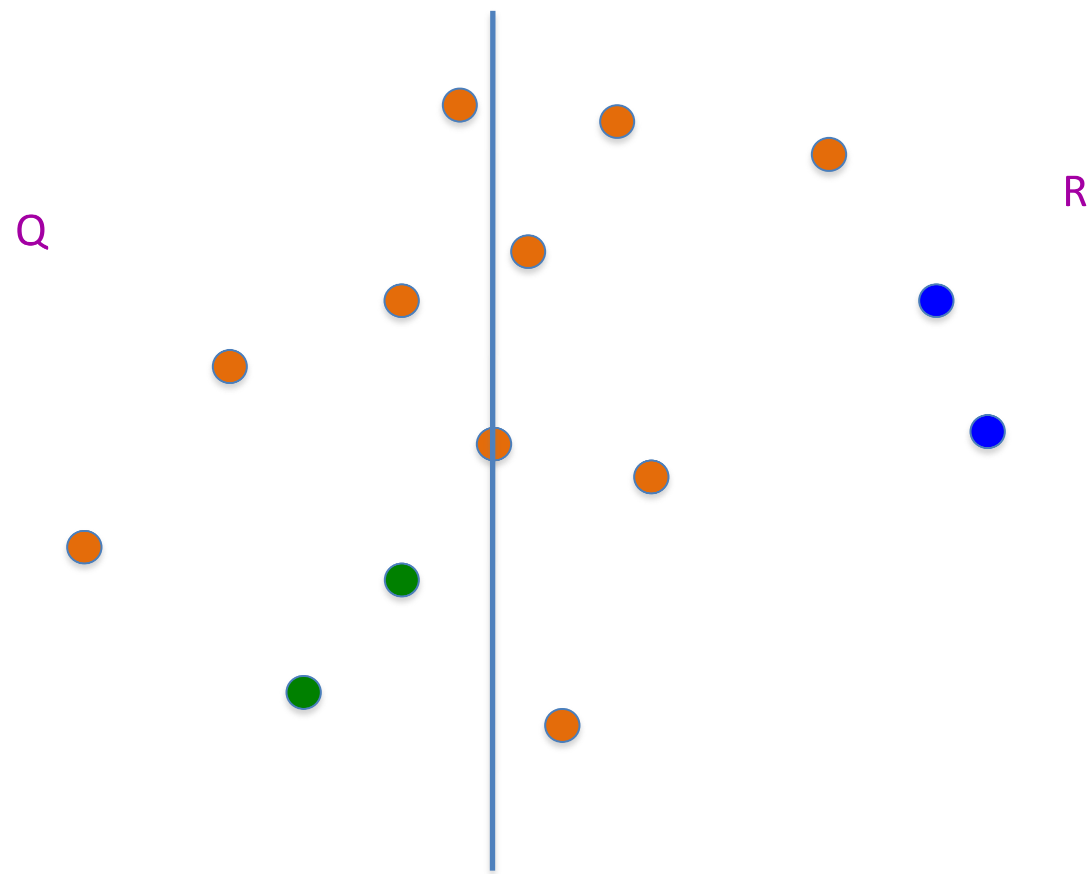
An easy case



All “crossing” pairs have distance > δ

δ = min (**blue**, **green**)

Life is not so easy though



δ = min (**blue**, **green**)