

Lecture 6

CSE 331

Sep 13, 2021

Please have a face mask on

Masking requirement



UR requires all students, employees and visitors – regardless of their vaccination status – to wear face coverings while inside campus buildings.

<https://www.buffalo.edu/coronavirus/health-and-safety/health-safety-guidelines.html>

2nd T/F poll up

poll @87

stop following

22 views

Actions

2nd T/F poll

Is the following statement true or false:

In every Stable Matching problem instance where a man m and woman w have each other as their least preferred partner, the following is true. There is no stable matching for the instance where (m, w) are matched.

(Note by a stable matching problem instance, we mean both the set of men and women as well as all the $2n$ preference lists.)

- True
- False

My office hour last Friday

Went through the process of writing a proof on the board!



We're not mind readers



If you need it, ask for help



Register your project groups

Deadline: Friday, Oct 1, 11:59pm

CSE 331 Syllabus Piazza Schedule Homeworks + Autolab **Project +** Support Pages + channel Sample Exams +

Forming groups

You form groups of size **exactly three (3)** for the project. Below are the various options.

Project Overview

Group signup form

- You have two choices in forming your group:

1. You can form your group on your own: i.e. you can submit the list of **EXACTLY three (3)** groups members in your group.

Note

Note that if you pick this option, your group needs to have **exactly THREE (3)** members. In particular, if your group has only two members you cannot submit as a group of size two. If you do not know many people in class, feel free to use piazza to look for the third group member.

2. You can submit *just your name*, and you will be assigned a random group among all students who take this second option. However, **note that if you pick this option you could end up in a group of size 2**. There will be at most two groups of size 2.

Submitting your group composition

Use this [Google form](#) to submit your group composition (the form will allow you to pick one of the two options above).

- You need to fill in the form for group composition by **11:59pm on Friday, October 1**.

Deadline is strict!

If you do not submit the form for group composition by the deadline, then you get a **zero for the entire project**.

HW 1 gets released this Wed

Wed, Sep 15	Gale Shapley algorithm outputs a stable matching    x^2	[KT, Sec 1.1] (HW 1 out)
Fri, Sep 17	Efficient algorithms and asymptotic analysis    x^2	[KT, Sec 1.1] Reading Assignment: Worst-case runtime analysis notes Reading Assignment: [KT, Sec 1.1, 2.1, 2.2, 2.4]
Mon, Sep 20	Runtime Analysis of Gale-Shapley algorithm    x^2	[KT, Sec 2.3]
Wed, Sep 22	Graph Basics    x^2	[KT, Sec 2.3, 3.1] (HW 2 out, HW 1 in)
Fri, Sep 24	Computing Connected Component    x^2	[KT, Sec 3.2] Reading Assignment: Care package on trees Reading Assignment: BFS by examples
Mon, Sep 27	Explore Algorithm    x^2	[KT, Sec 3.2]
Wed, Sep 29	Runtime Analysis of BFS algorithm    x^2	[KT, Sec 3.3] (HW 3 out, HW 2 in)
Fri, Oct 1	More graph stuff    x^2	[KT, Sec 3.3, 3.6] (Project Team Composition Due) Reading Assignment: [KT, Sec 3.3, 3.4, 3.5, 3.6] Reading Assignment: Care package on topological ordering
Mon, Oct 4	Interval Scheduling Problem    x^2	[KT, Sec 4.1]
Wed, Oct 6	Greedy Algorithm for Interval Scheduling    x^2	[KT, Sec 4.1] (HW 3 in) (Project out) Reading Assignment: [KT, Sec 4.1, 4.2]
Fri, Oct 8	Shortest Path Problem    x^2	[KT, Sec 4.4] (Quiz 1) Reading Assignment: Care package on minimizing maximum lateness
Mon, Oct 11	Mid-term exam: I	
Wed, Oct 13	Mid-term exam: II	(HW 4 out)

Questions/Comments?



Reading Assignment - I

note @00    - stop following **4** views

Reading Assignment: Asymptotic Analysis

As one of the changes made in F19, we will assume that y'all are familiar with asymptotic analysis and not spend reviewing it in any detail during the lectures. In case you are not that comfortable with asymptotic analysis and/or want to review the material, please read through the asymptotic analysis care package:

<http://www-student.cse.buffalo.edu/~atri/cse331/support/care-package/asymptotics/index.html>

We will need this either the middle of lecture on Wednesday or in the Friday lecture.

[lectures](#)

edit good note | 

Updated 41 seconds ago by Adri Rutka

Reading Assignment - II

note @00

stop following 3 viewers

Reading Assignment: Pigeonhole principle

Another reading assignment for this week (here is the other one: [@110](#)). Please go through this support page on pigeonhole principle--

<http://www-student.cse.buffalo.edu/~atr/csa331/support/pigeon/index.html>

It's actually a very simple result that turns out to be surprisingly powerful. We'll use this in the Mon/Wed lecture.

pictures

edit

good note

Updated 37 seconds ago by Aki Ruzsa

Stable Marriage problem

Set of men M and women W

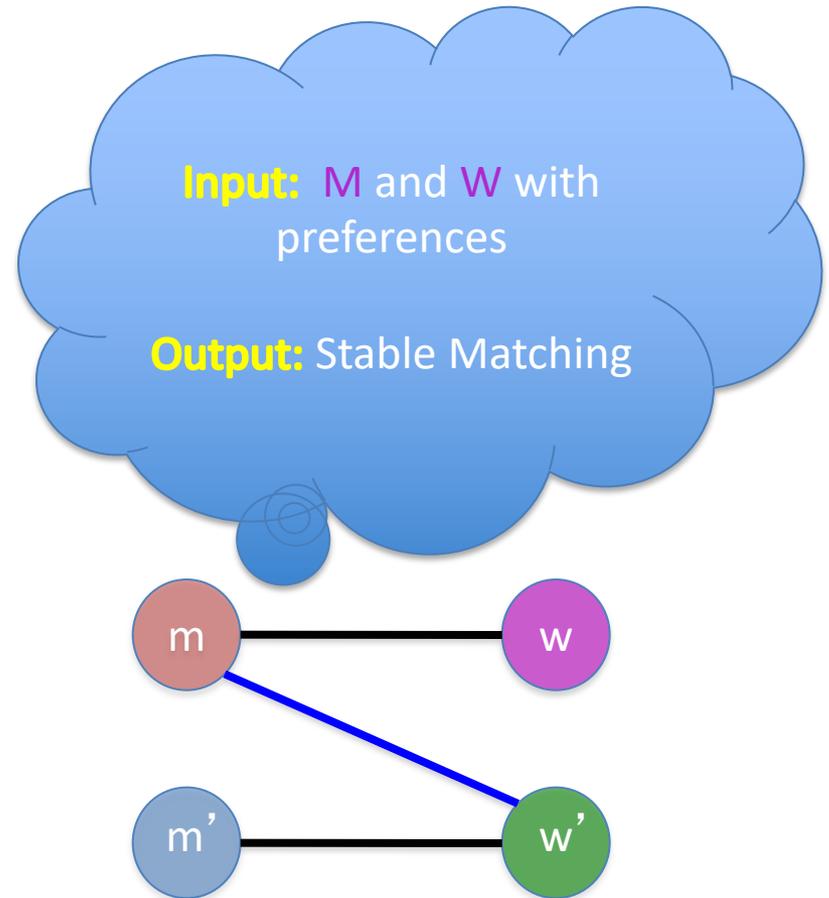
Preferences (ranking of potential spouses)

Matching (no polyandry/gamy in $M \times W$)

Perfect Matching (everyone gets married)

Instability

Stable matching = perfect matching + no instability



Two Questions

Does a stable marriage always exist?

If one exists, how quickly can we compute one?

The naïve algorithm

Incremental algorithm to produce all $n!$ perfect matchings?

Go through all possible perfect matchings S

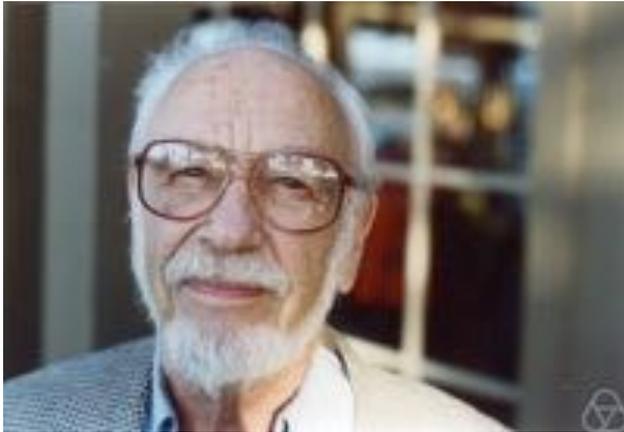
If S is a stable matching

then Stop



Else move to the next perfect matching

Gale-Shapley Algorithm



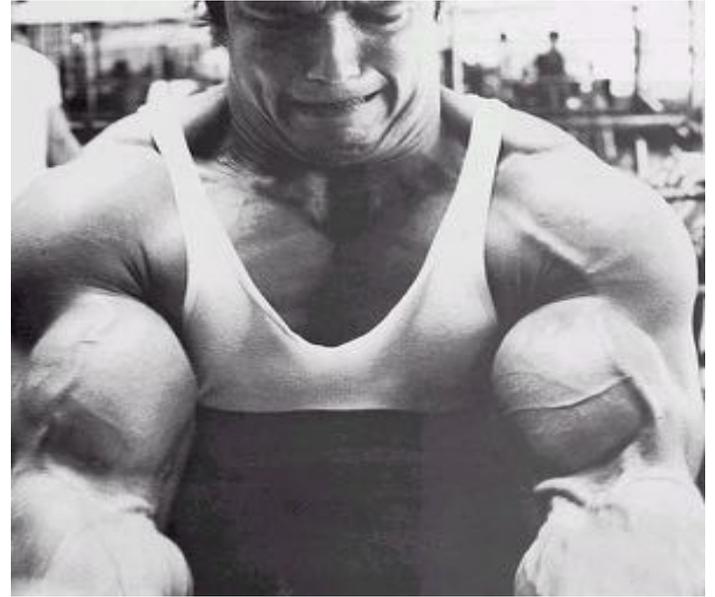
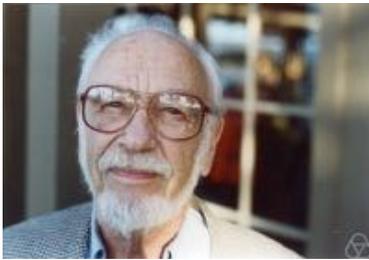
David Gale



Lloyd Shapley

$O(n^2)$ algorithm

Moral of the story...



Questions/Comments?



Rest of today's agenda

GS algorithm

Run of GS algorithm on an instance

Prove correctness of the GS algorithm

Back to the board...



Gale-Shapley Algorithm

Initially all men and women are **free**

While there exists a free woman who can propose

Let w be such a woman and m be the best man she has not proposed to

w proposes to m

If m is free

(m,w) get **engaged**

Else (m,w') are engaged

If m prefers w' to w

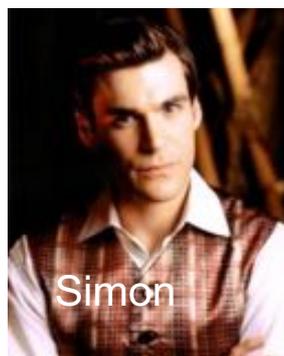
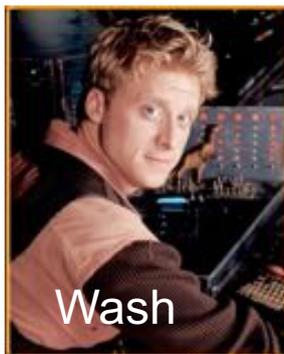
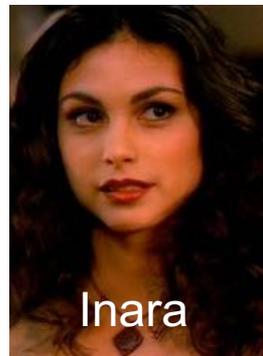
w remains **free**

Else

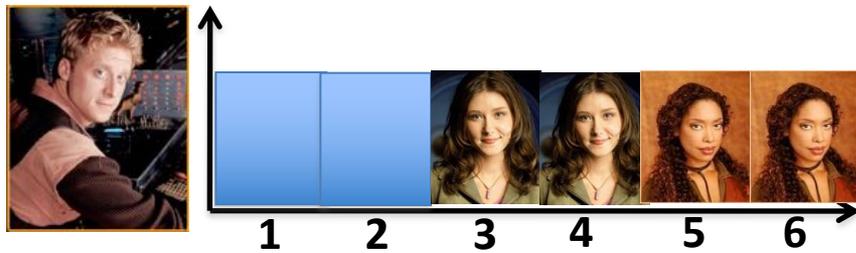
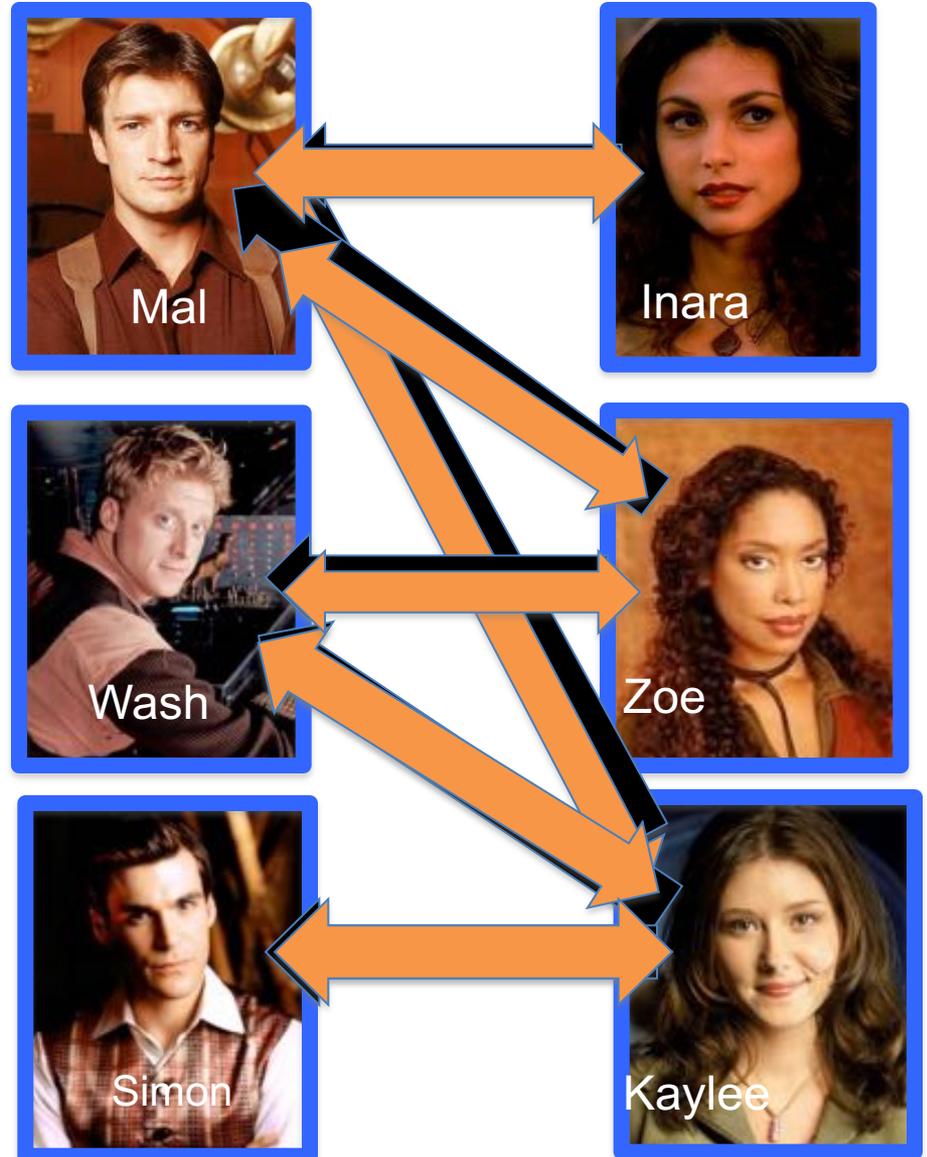
(m,w) get **engaged** and w' is **free**

Output the engaged pairs as the final output

Preferences



GS algorithm: Firefly Edition



Observation 1

Initially all men and women are **free**

While there exists a free woman who can propose

Let w be such a woman and m be the best man she has not proposed to

w proposes to m

If m is free

(m,w) get **engaged**

Else (m,w') are engaged

If m prefers w' to w

w remains **free**

Else

(m,w) get **engaged** and w' is **free**

Once a man gets engaged, he remains engaged (to “better” women)

Output the engaged pairs as the final output

Observation 2

Initially all men and women are **free**

While there exists a free woman who can propose

Let w be such a woman and m be the best man she has not proposed to

w proposes to m

If m is free

(m,w) get **engaged**

Else (m,w') are engaged

If m prefers w' to w

w remains **free**

Else

(m,w) get **engaged** and w' is **free**

If w proposes to m after m' , then she prefers m' to m

Output the set S of engaged pairs as the final output

Questions/Comments?



Why bother proving correctness?

Consider a variant where any free man **or** free woman can propose

Is this variant any different? Can you prove it?

GS' does not output a stable marriage

