Lecture 8

CSE 331

Sep 16, 2021

Please have a face mask on

Masking requirement

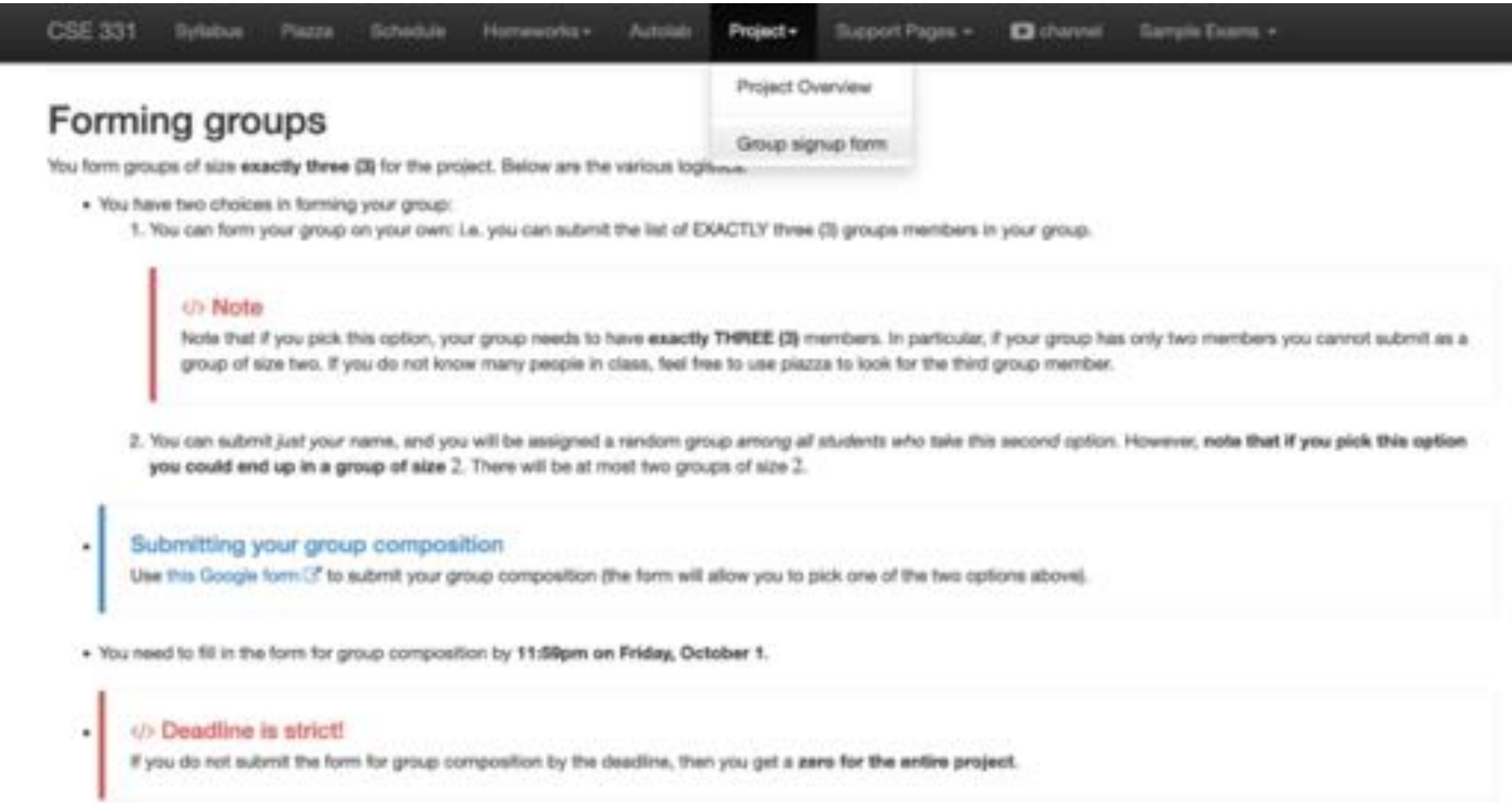
UR requires all students, employees and visitors – regardless of their vaccination status – to wear face coverings while inside campus buildings.

https://www.buffalo.edu/coronavirus/health-and-safety/health-safety-guidelines.html

If you need it, ask for help

Register your project groups

Deadline: Friday, Oct 1, 11:59pm



CSE 331 Syllabus Piazza Schedule Homeworks Autolab Project Support Pages channel Sample Exams

Project Overview

Group signup form

Forming groups

You form groups of size **exactly three (3)** for the project. Below are the various logi

- You have two choices in forming your group:
  1. You can form your group on your own: i.e. you can submit the list of EXACTLY three (3) groups members in your group.

     Note

     Note that if you pick this option, your group needs to have **exactly THREE (3)** members. In particular, if your group has only two members you cannot submit as a group of size two. If you do not know many people in class, feel free to use piazza to look for the third group member.

  2. You can submit *just your name*, and you will be assigned a random group *among all students who take this second option*. However, **note that if you pick this option you could end up in a group of size** 2. There will be at most two groups of size 2.

- **Submitting your group composition**

  Use this Google form to submit your group composition (the form will allow you to pick one of the two options above).

- You need to fill in the form for group composition by **11:59pm on Friday, October 1.**

- **Deadline is strict!**

  If you do not submit the form for group composition by the deadline, then you get a **zero for the entire project.**

Questions/Comments?

Gale-Shapley Algorithm

Intially all men and women are free

While there exists a free woman who can propose

Let w be such a woman and m be the best man she has not proposed to

w proposes to m

If m is free

(m,w) get engaged

Else (m,w') are engaged

If m prefers w' to w

w remains free

Else

(m,w) get engaged and w' is free

Output the engaged pairs as the final output

The Lemmas

Lemma 1: The GS algorithm has at most n^2 iterations

Lemma 2: S is a perfect matching

Lemma 3: S has no instability

Proof Details of Lemma 1

Gale Shapley algorithm terminates

This page collects material from Fall 17 incarnation of CSE 331, where we proof details for the claim that the Gale-Shapley algorithm terminates in $O(n^2)$ iterations.

Where does the textbook talk about this?

`Section 1.1` in the textbook has the argument (though not in as much detail as below).

Fall 2017 material

Here is the lecture video (it starts from the part where we d the proof details):

Questions/Comments?

Proof technique de jour

Proof by contradiction

Assume the negation of what you want to prove

After some reasoning

Source: 4simpsons.wordpress.com

Two obervations

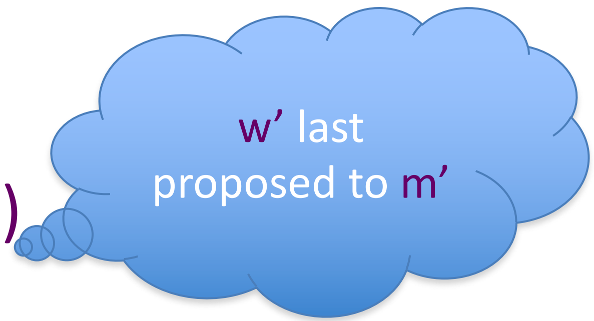
Obs 1: Once m is engaged he keeps getting engaged to “better” women

Obs 2: If w proposes to m’ first and then to m (or never proposes to m) then she prefers m’ to m

Proof of Lemma 3

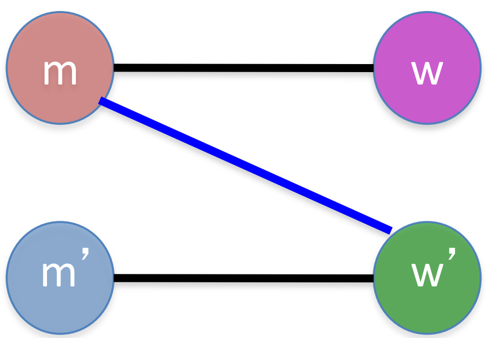
By contradiction

Assume there is an instability (m,w’)



m prefers w’ to w

w’ prefers m to m’



Contradiction by Case Analysis

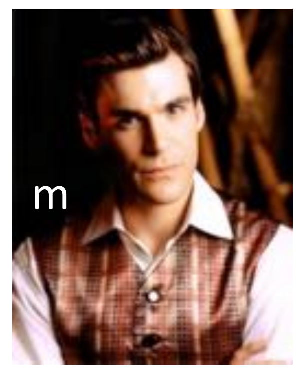
Depending on whether w’ had proposed to m or not

Case 1: w’ never proposed to m

w’ prefers m’ to m

By Obs 2

Assumed w’ prefers m to m’



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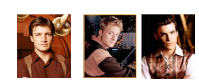


Case 2: w’ had proposed to m

Case 2.1: m had accepted w’ proposal

m is finally engaged to w

4simpsons.wordpress.com

Thus, m prefers w to w’

By Obs 1

Case 2.2: m had rejected w’ proposal

m was engaged to w’’ (prefers w’’ to w’)

By Algo def

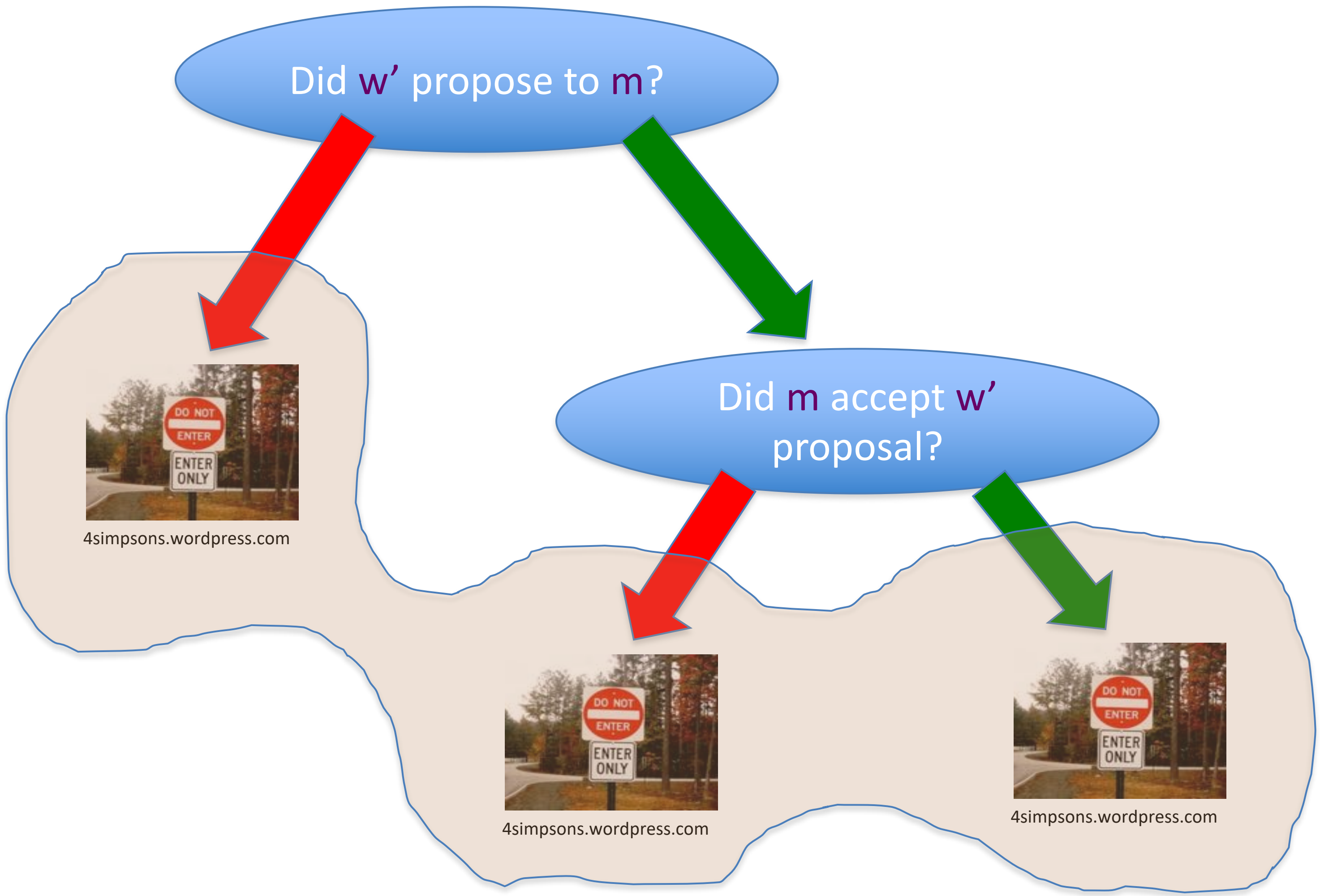
m is finally engaged to w (prefers w to w’’)

By Obs 1

m prefers w to w’

4simpsons.wordpress.com

Overall structure of case analysis



Questions?

Extensions

Fairness of the GS algorithm

Different executions of the GS algorithm

Main Steps in Algorithm Design

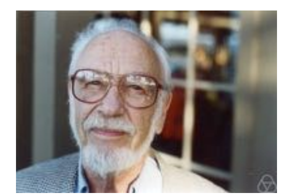
NRMP
National Resident Matching Program

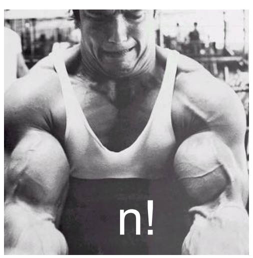
Problem Statement

↓

Problem Definition

↓





Algorithm

↓

"Implementation"

↓

Analysis

Correctness Analysis

Definition of Efficiency

An algorithm is efficient if, when implemented, it runs quickly on real instances

Implemented where?



Platform independent definition

What are real instances?

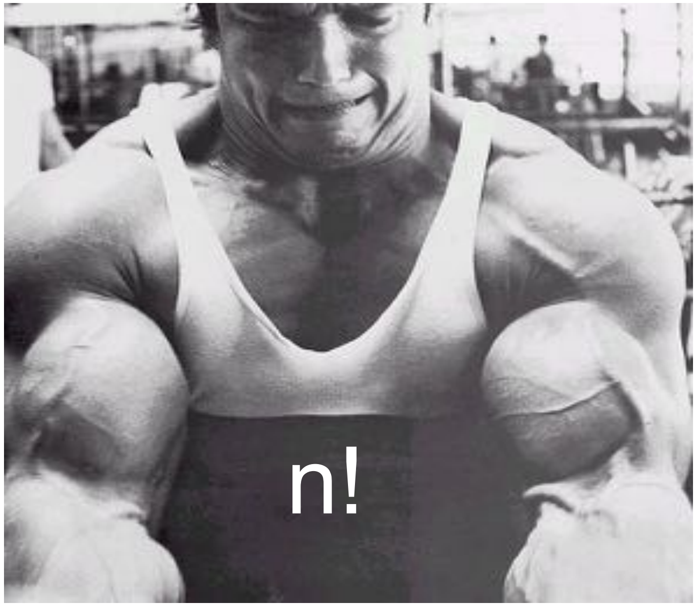
Worst-case Inputs

$N = 2n^2$ for SMP

Efficient in terms of what?

Input size N

Definition-II



Analytically better than brute force

How much better? By a factor of 2?

Definition-III

Should scale with input size

If N increases by a constant factor, so should the measure

Polynomial running time

At most $c \cdot N^d$ steps ($c>0$, $d>0$ absolute constants)

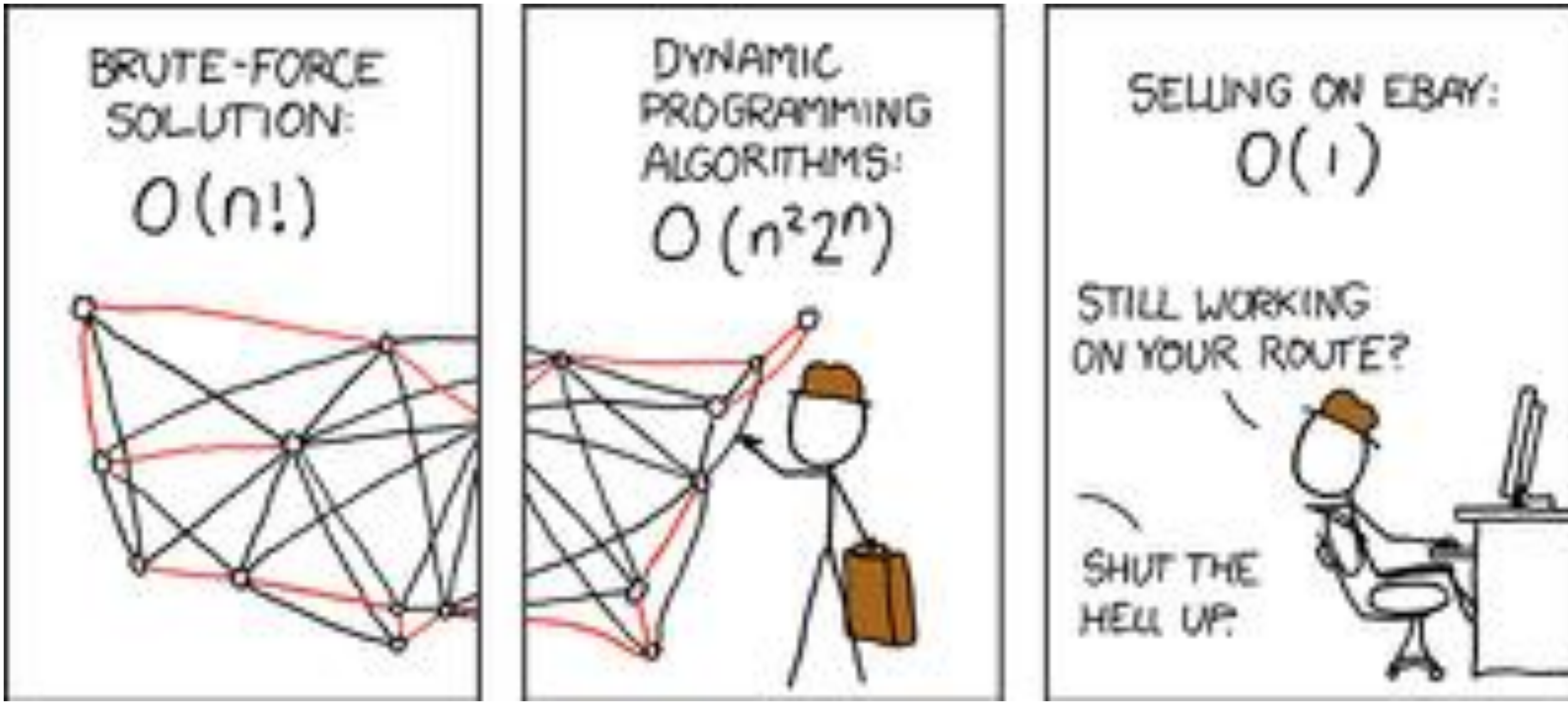
Step: "primitive computational step"

More on polynomial time

Problem centric tractability

Can talk about problems that are not efficient!

Asymptotic Analysis



Travelling Salesman Problem

(http://xkcd.com/399/)

Reading Assignment for today

note @89

stop following 70 views

Reading Assignment: Asymptotic Analysis

As one of the changes made in F19, we will assume that y'all are familiar with asymptotic analysis and not spend reviewing it in any detail during the lectures. In case you are not that comfortable with asymptotic analysis and/or want to review the material, please read through the asymptotic analysis care package:

http://www-student.cse.buffalo.edu/~atri/cse331/support/care-package/asymptotics/index.html

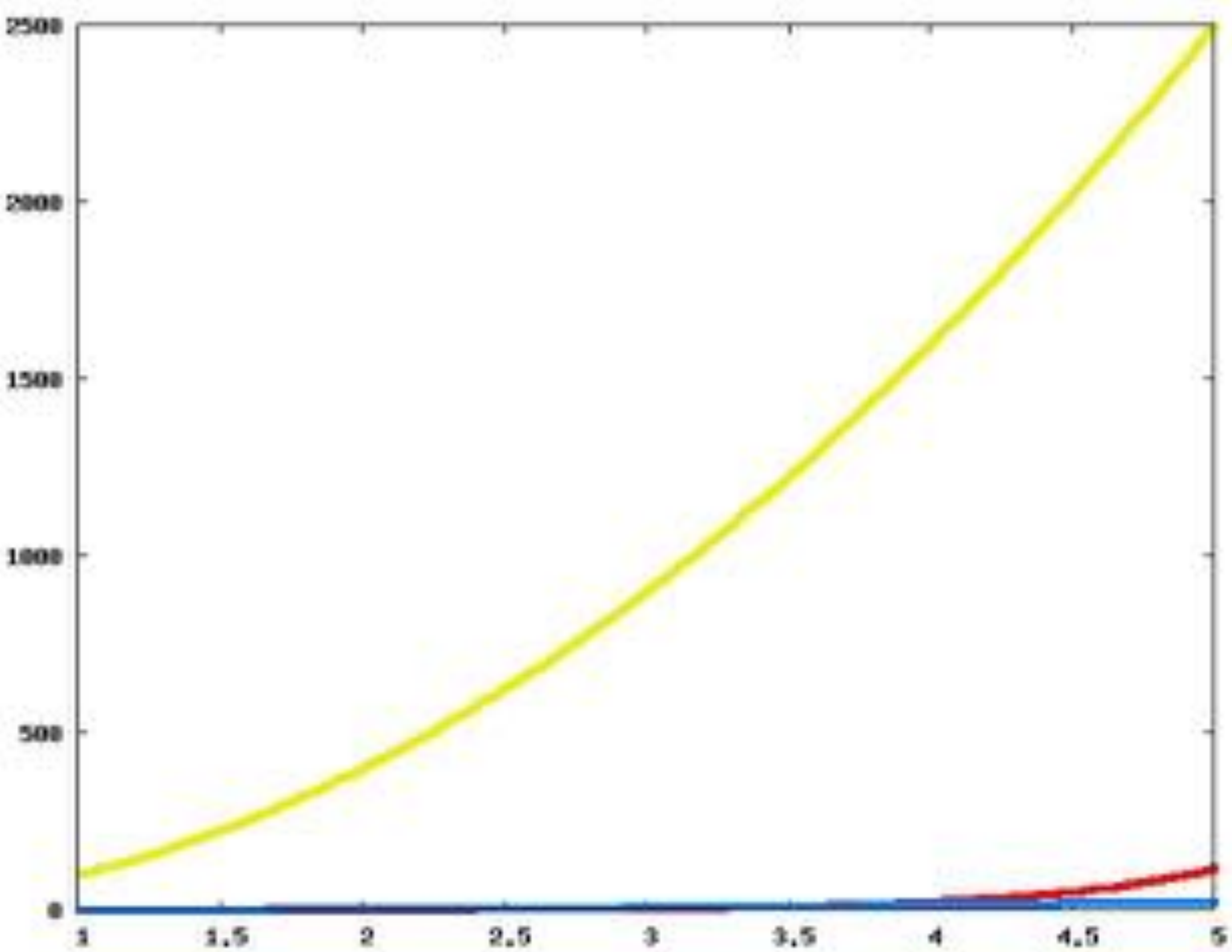
We will need this either the middle of lecture on Wednesday or in the Friday lecture.

lectures

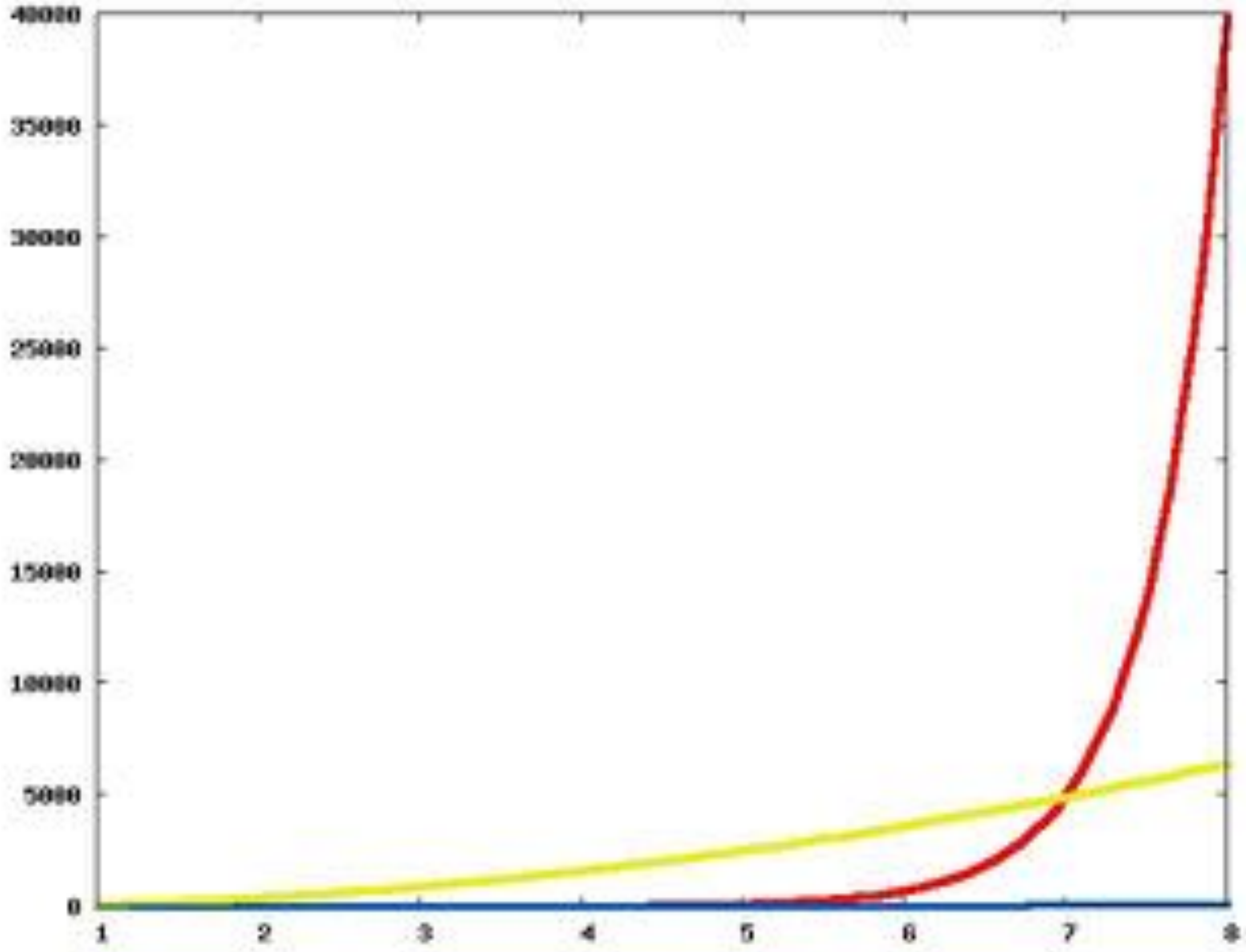
edit good note | 0

Updated 3 days ago by Atri Rudra

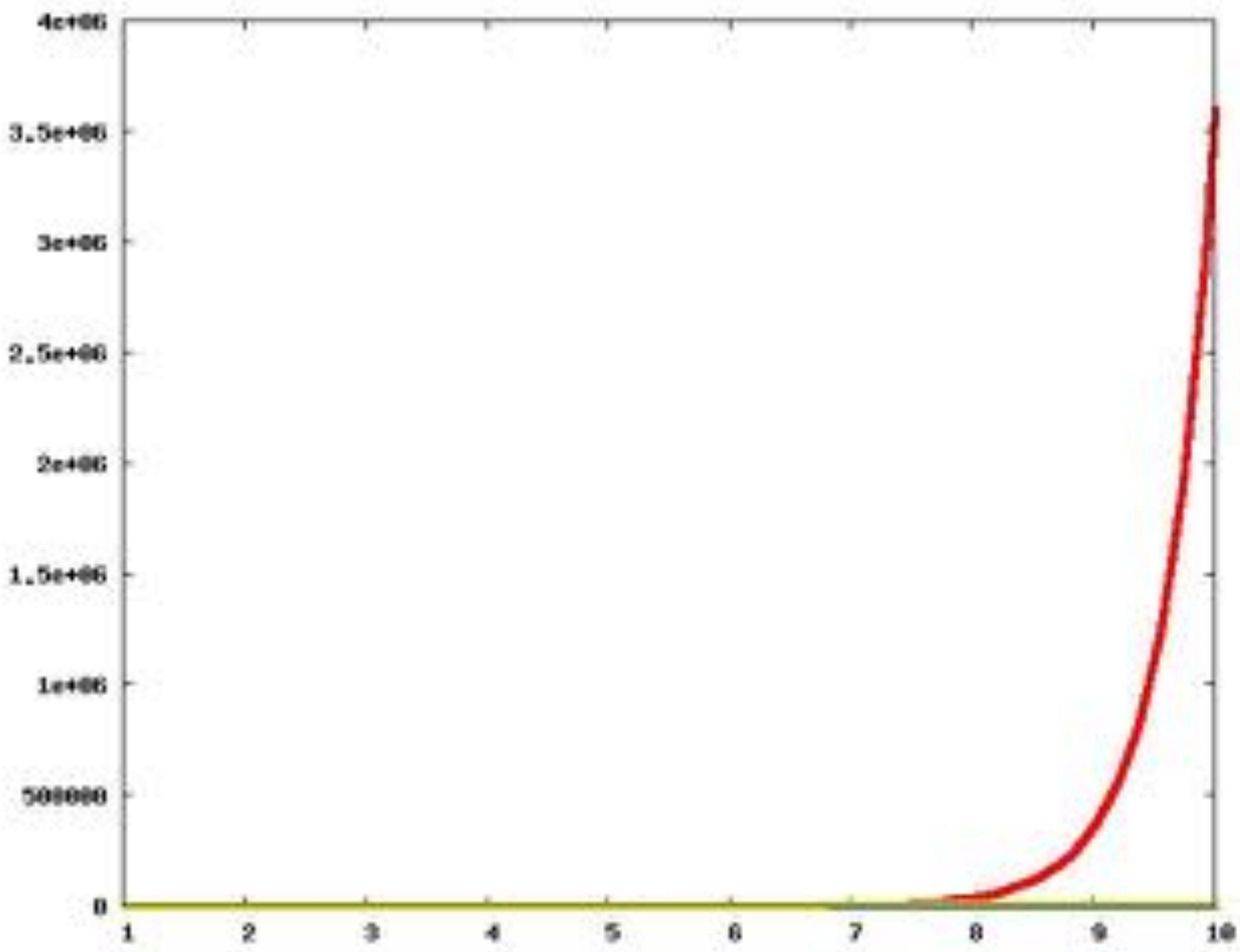
Which one is better?



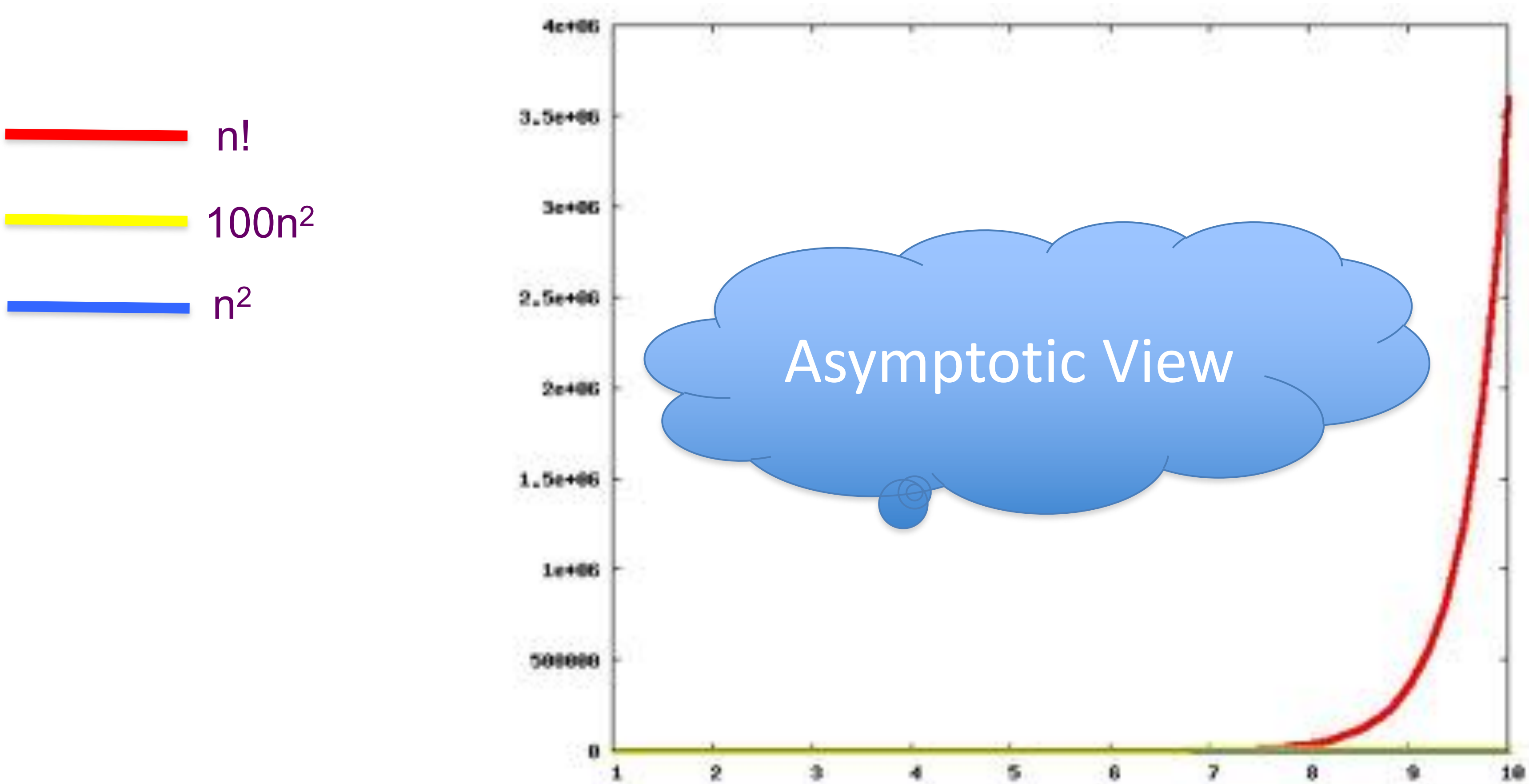
Now?



And now?



The actual run times



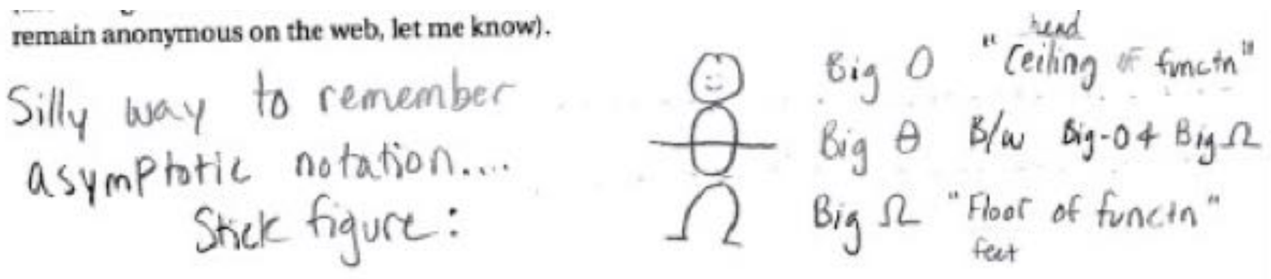
Asymptotic Notation

$\leq$ is O with glasses

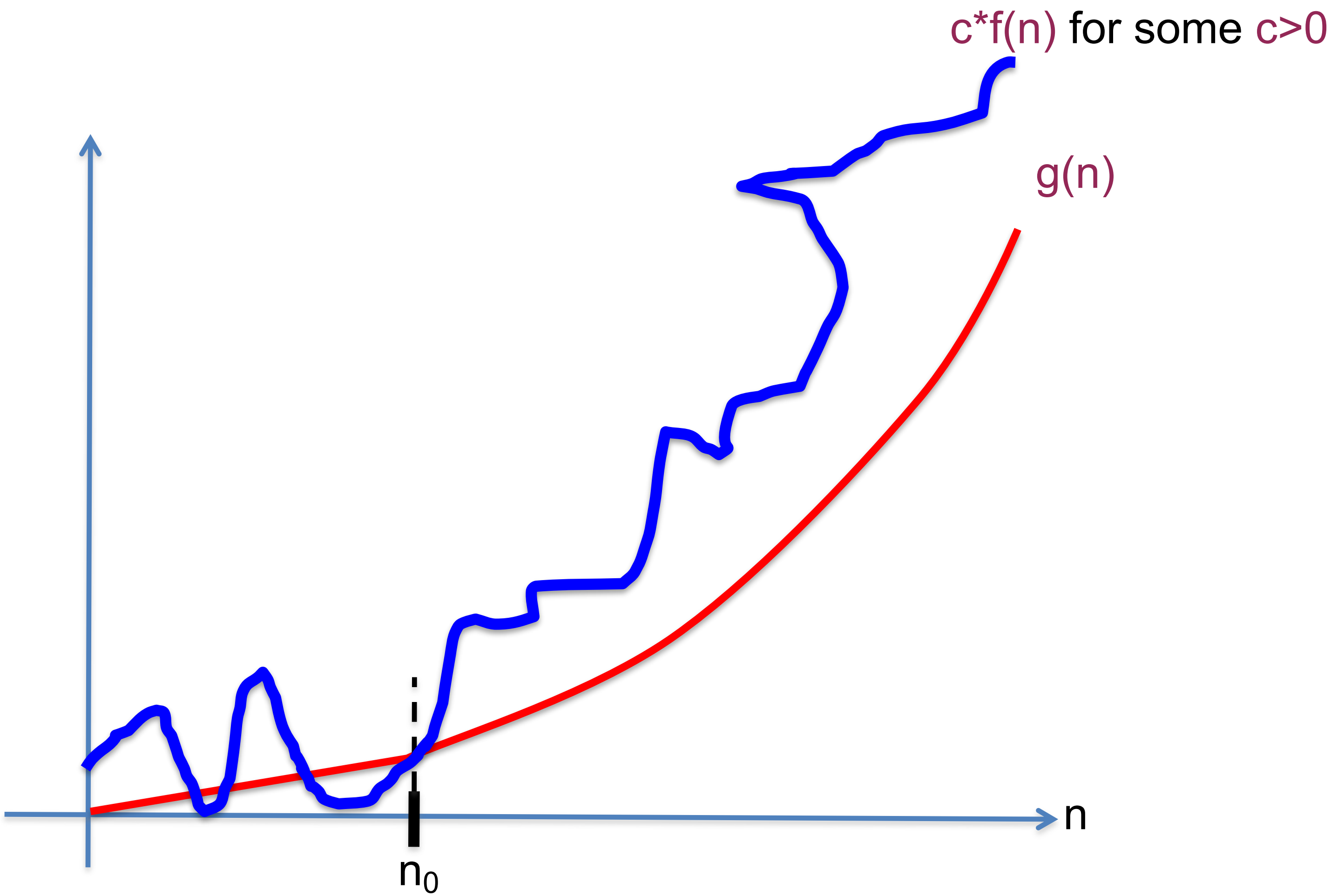
$\geq$ is Ω with glasses

$=$ is Θ with glasses

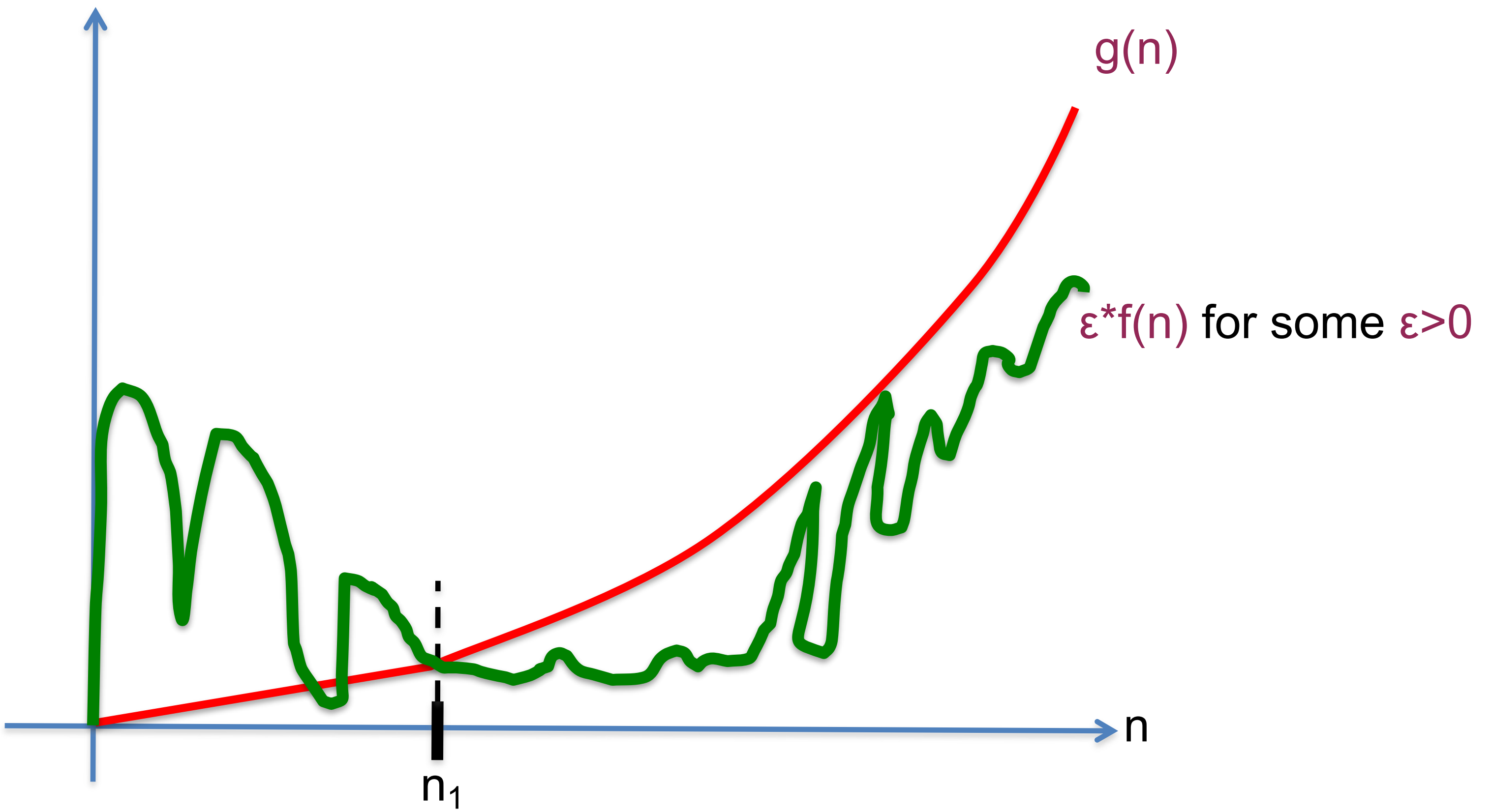
Another view



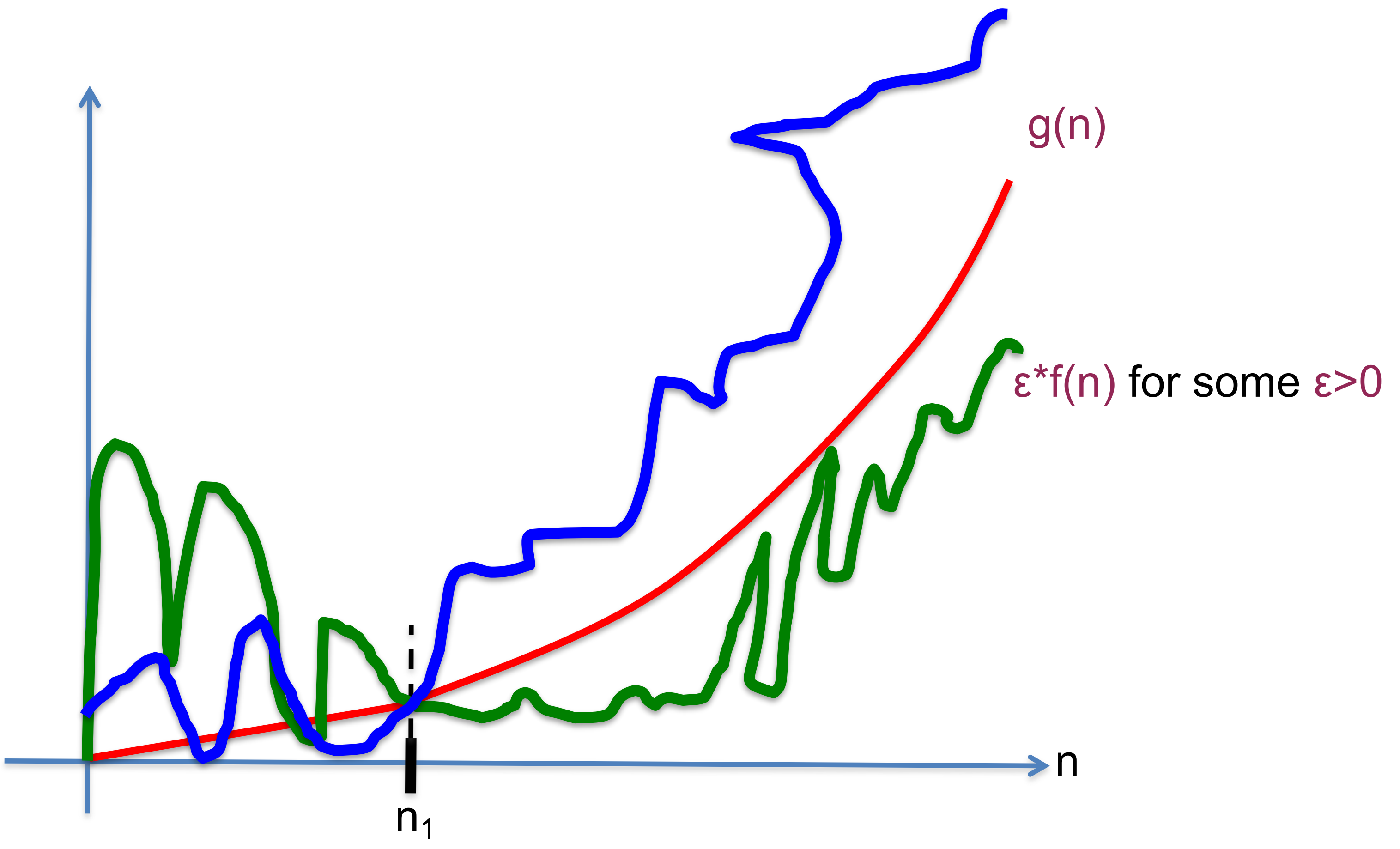
g(n) is O(f(n))



g(n) is Ω(f(n))



g(n) is Θ(f(n))



Properties of O (and Ω)

Transitive

g is O(f) and f is O(h) then
g is O(h)

Step 1 // O(n) time
Step 2 // O(n) time

Overall:
O(n) time

Additive

g is O(h) and f is O(h) then
g+f is O(h)

Multiplicative

g is $O(h_1)$ and f is $O(h_2)$ then
g*f is $O(h_1 * h_2)$

Overall:
$O(n^2)$ time

While (loop condition) // $O(n^2)$ iterations
Stuff happens // O(1) time

Another Reading Assignment

CSE 331 Support Pages

Analyzing the worst-case runtime of an algorithm

Some notes on strategies to prove Big-Oh and Big-Omega bounds on runtime of an algorithm.

The setup

Let $\mathcal{A}$ be the algorithm we are trying to analyze. Then we will define $T(N)$ to be the worst-case run-time of $\mathcal{A}$ over all inputs of size N. Slightly more formally, let $t_{\mathcal{A}}(\mathbf{x})$ be the number of steps taken by the algorithm $\mathcal{A}$ on input $\mathbf{x}$. Then

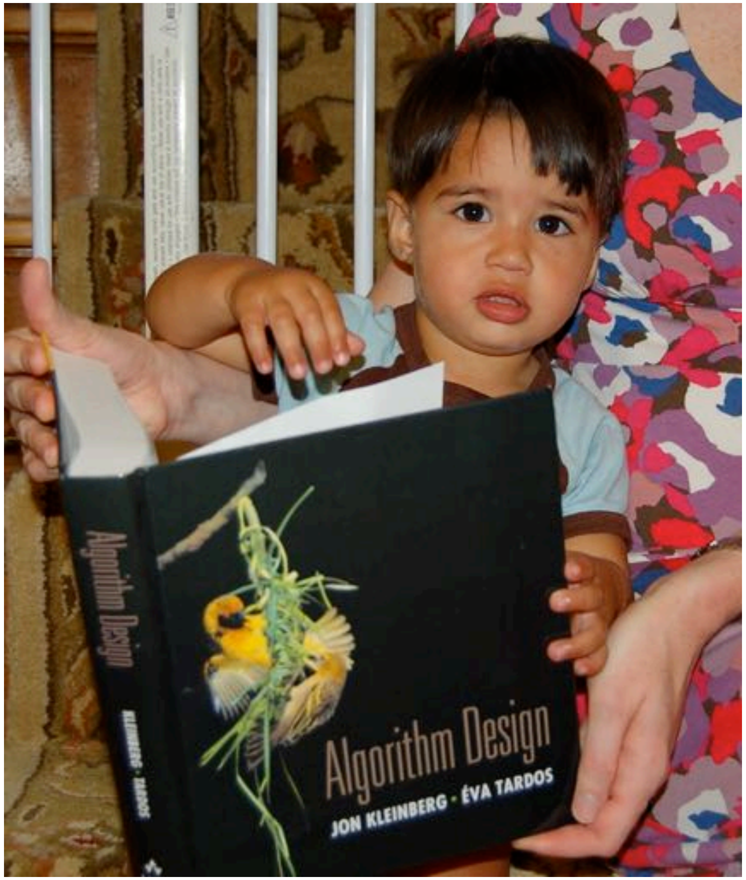
$$T(N) = \max_{\mathbf{x}: \mathbf{x} \text{ is of size } N} t_{\mathcal{A}}(\mathbf{x}).$$

In this note, we present two useful strategies to prove statements like $T(N)$ is $O(g(N))$ or $T(N)$ is $\Omega(h(N))$. Then we will analyze the run time of a very simple algorithm.

Preliminaries

We now collect two properties of asymptotic notation that we will need in this note (we saw these in class today).

Reading Assignments



Sections 1.1, 1.2, 2.1, 2.2 and 2.4 in [KT]

Questions?

Rest of today's agenda

Analyzing the run time of the GS algo

Gale-Shapley Algorithm

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Implementation Steps

How do we represent the input?

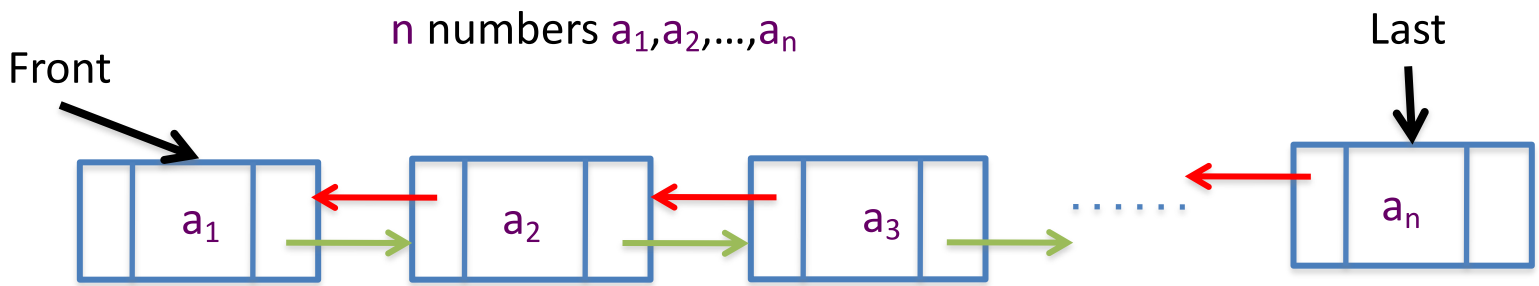
How do we find a free woman w?

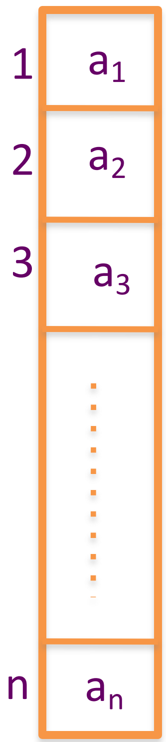
How would w pick her best unproposed man m?

How do we know who m is engaged to?

How do we decide if m prefers w' to w?

Arrays and Linked Lists





	Array	Linked List
Access ith element	O(1)	O(i)
Is e present?	O(n) (O(log n) if sorted)	O(n)
Insert an element	O(n)	O(1) given pointer
Delete an element	O(n)	O(1) given pointer
Static vs Dynamic	Static	Dynamic